

Dawson College

Physics 203-NYB-05 Electricity & Magnetism

*Sample* Final Examination

This exam is divided into two parts:

**Part I: Problems (10 marks each)** Solve all six problems. Show all of your work, clearly and in order, to receive full marks. If you use a formula not given on the formula sheet, a derivation must be shown.

**Part II: Multiple Choice Questions (2 marks each)** Answer all twenty questions. Circle the best response from the choices given. If your final selection is unclear you will not be given the marks. No marks will be awarded for diagrams, calculations, or reasoning.

Additional instructions:

1. The best way to use this practice exam is to set aside three hours (the time you will have in the real exam) and do it. Look at the answers and solutions only after you tried writing the whole sample exam.
2. Answer and solutions are found at the end of this document.
3. Use  $c = 3.00 \times 10^8$  m/s for the value of the speed of light, if you need it.

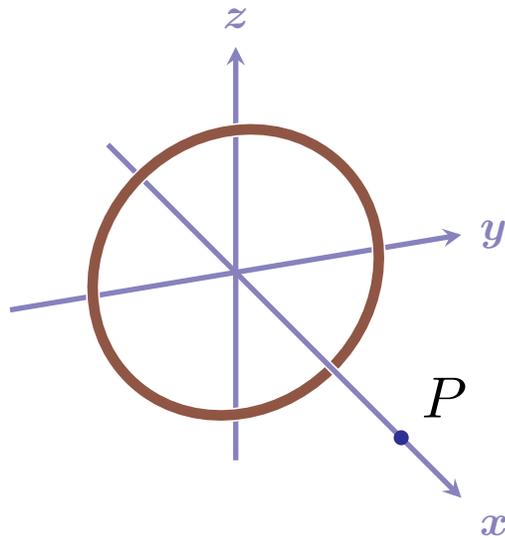
Good luck! ☺

## Part I: Problems (10 marks each)

Solve all six problems. Show all of your work, clearly and in order, to receive full marks. If you use a formula not given on the formula sheet, a derivation must be shown.

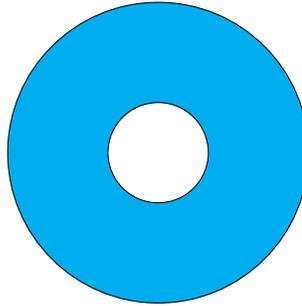
---

1. Pictured below is uniformly charged ring in the  $yz$ -plane, centered at the origin. The ring has radius  $R$  and linear charge density  $\lambda$ .



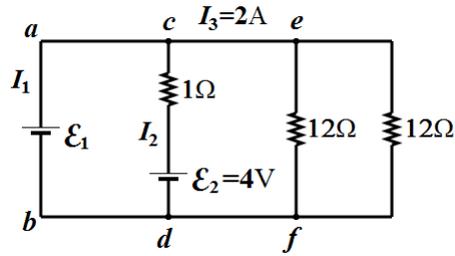
- (a) (7pt) Integrate contributions from the ring to determine the expression for the electrical potential at a position  $P$  on the  $x$ -axis. (Use  $V = 0$  V infinitely far away from the ring.)
- (b) (3pt) What is the electric field vector at that position? State all three of its components. (Hint: you do not need to integrate, again. Use the result of part (a).)

2. A hollow sphere made from a non-conducting material is shown below in cross-section. The inner radius is  $R_1$ , and the outer radius is  $R_2$ . The material is charged uniformly  $\rho$ .



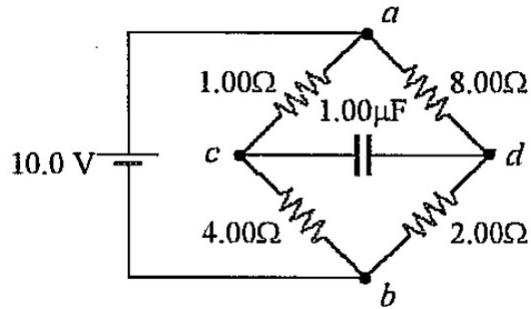
- (a) (7pt) What is the magnitude (measured in N/C) and the direction (away from or towards the center) of the electric field at a distance  $r$  from the center of the sphere, for values  $R_1 < r < R_2$ ? (For full marks you are required to use Gauss' Law only to solve this first part of the problem. Do all this work symbolically – don't use the values of part (b) for this part.)
- (b) (3pt) The inner radius is  $R_1 = 1.00$  cm, and the outer radius is  $R_2 = 3.00$  cm. The material is charged uniformly  $\rho = -1.07$  nC/m<sup>3</sup>. If the electric potential is 0 V infinitely far away, what is the electric potential at the outer surface of the sphere?

3. Below is a circuit with four branches. The currents  $I_1$  and  $I_2$ , and the emf  $\mathcal{E}_1$  are unknown. The current flowing from  $c$  to  $e$  is 2 A.



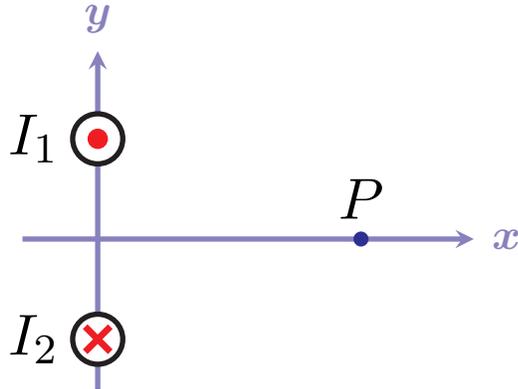
- (2pt) What is the current flowing through each of the  $12\ \Omega$  resistors?
- (2pt) What is the value of the emf  $\mathcal{E}_1$ ?
- (4pt) What are the currents  $I_1$  and  $I_2$ , and in what directions do they flow?
- (2pt) Is the emf  $\mathcal{E}_2$  providing energy or is it absorbing energy? What is the power it is providing or absorbing?

4. Below is an  $RC$  circuit. Initially, before the battery is connected, the capacitor  $C$  is uncharged.



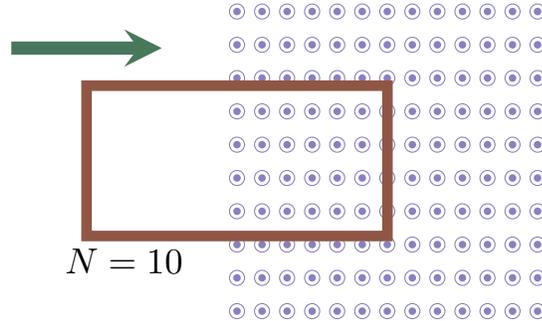
- (4pt) Immediately after the battery is connected what is the current flowing through the capacitor? (State the value of the current and its direction of flow.)
- (4pt) After a very long time, what will be the charge on the capacitor?
- (2pt) If the battery is then disconnected from the circuit, after how much time will the charge on the capacitor decrease to 1/10 of its initial value?

5. Two long, straight, parallel current-carrying wires are shown below. The wires are parallel to the  $z$ -axis (the  $+z$ -axis points out of the page). The current in the wire above the origin is flowing in the  $+z$ -direction ( $I_1 = 3.2\text{ A}$ , out of the page), while the current in the wire below the origin is flowing in the  $-z$ -direction ( $I_2 = 3.2\text{ A}$ , into the page). The wire above the origin passes through  $y = +13.3\text{ cm}$ , the wire below the origin passes through  $y = -13.3\text{ cm}$ , and the point  $P$  is at  $x = +35.0\text{ cm}$ .



- (5pt) Find all three components of the magnetic field vector at the point  $P$  on the  $x$ -axis.
- (2pt) If an electron were traveling in the  $+z$ -direction at  $5200\text{ m/s}$  what would be the magnitude and direction of the magnetic force acting on it as it passed through  $P$ ?
- (3pt) What electric field (state all three components) would we need to apply to the electron in part (b) so that the net force acting on it were zero?

6. There is a rectangular conducting coil, measuring 40.0cm by 20.0cm, with resistance 50.0ohm has 10 turns, as shown below. The coil moves at a constant speed of 50.0 cm/s from a region where the magnetic field is zero and into a region where the field is 2.00 T pointed along the  $+z$ -axis.



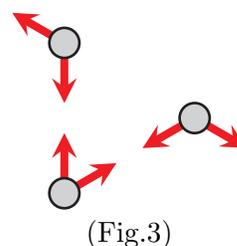
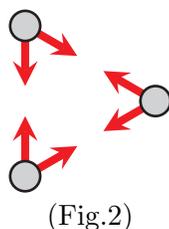
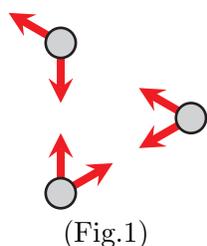
While the coil is in the process of entering the magnetic field (and neither full in nor full out of the field):

- (4pt) Determine the magnitude and direction (as viewed from above) of the induced current in the coil.
- (2pt) Determine the magnitude and direction of the magnetic force that acts on the coil.
- (4pt) From the moment that the coil begins to enter the field until the moment that the coil has fully entered the field what will be the total amount of energy dissipated by the coil's resistance?

## Part II: Multiple Choice Questions (2 marks each)

Answer all twenty questions. Circle the best response from the choices given. If your final selection is unclear you will not be given the marks. No marks will be awarded for diagrams, calculations, or reasoning.

1. Which of the following diagrams represent the forces that three charged objects might exert on each other?



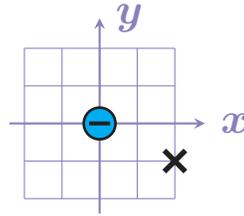
- (a) Fig.1.  
(b) Fig.2.  
(c) Fig.3.  
(d) Fig.1 and Fig.3, but not Fig.2.  
(e) All three.
2. The force acting on an electron is  $4.337 \times 10^{-16}$  N along the  $+y$ -axis. What is the magnitude and direction of the electric field vector at the electron's position?:
- (a)  $6.95 \times 10^{-35}$  N/C along the  $-y$ -axis.  
(b)  $4.76 \times 10^{+14}$  N/C along the  $-y$ -axis.  
(c)  $2.71 \times 10^{+3}$  N/C along the  $-y$ -axis.  
(d)  $2.71 \times 10^{+3}$  N/C along the  $+y$ -axis.  
(e) Defined only for a positive charge.

3. Two positively charged objects are on the  $x$ -axis, as shown below. The object on the left has a larger charge than the object on the right.



On the  $x$ -axis there is a position where  $\vec{E} = \vec{0}$  N/C. This position is

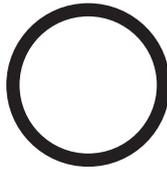
- (a) to the left of the larger charge.
  - (b) between the two charges, but closer to the larger charge.
  - (c) between the two charges, exactly in the middle.
  - (d) between the two charges, but closer to the smaller charge.
  - (e) to the right of the smaller charge.
4. A  $-3$  nC charge is at the origin, as shown below.



What are the components  $(E_x, E_y)$  of the electric field at the position "X", measured in N/C?

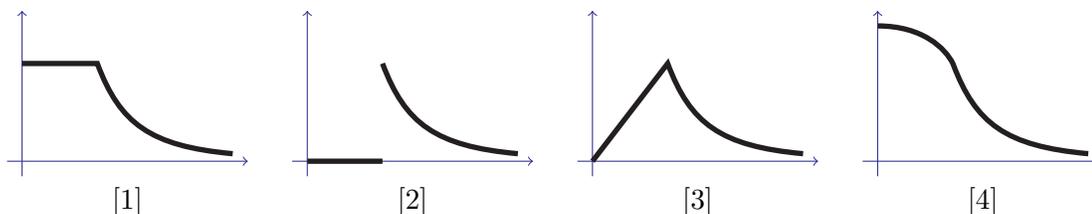
- (a)  $(+241, -482)$ .
  - (b)  $(-241, +482)$ .
  - (c)  $(+482, +241)$ .
  - (d)  $(-482, -241)$ .
  - (e)  $(-482, +241)$ .
5. A positively charged particle is in a uniform magnetic field. As it moves along the  $-x$ -axis ( $\vec{v} = -v\hat{i}$ ) the magnetic force it experiences points along the  $-z$ -axis ( $\vec{F}_m = -F\hat{k}$ ). The magnetic field vector points along
- (a) the  $+x$ -axis ( $+\hat{i}$ ).
  - (b) the  $+y$ -axis ( $+\hat{j}$ ).
  - (c) the  $+z$ -axis ( $+\hat{k}$ ).
  - (d) the  $-y$ -axis ( $-\hat{j}$ ).
  - (e) some other direction, not listed.

6. A proton has velocity  $\vec{v} = (-300 \text{ m/s})\hat{i} + (+400 \text{ m/s})\hat{j}$ . It is moving through a uniform magnetic field  $\vec{B} = (0.5 \text{ T})\hat{i} + (+0.5 \text{ T})\hat{j}$ . The magnetic force acting on the proton is:
- (a)  $+8.01 \times 10^{-18} \text{ N } \hat{k}$ .
  - (b)  $-8.01 \times 10^{-18} \text{ N } \hat{k}$ .
  - (c)  $+5.61 \times 10^{-17} \text{ N } \hat{k}$ .
  - (d)  $-5.61 \times 10^{-17} \text{ N } \hat{k}$ .
  - (e) zero.
7. A conducting loop and a long straight wire are shown.



- There is a current along the wire and a current around the loop. If the magnetic field is zero at the center of the loop then direction of current flow in the wire and in the loop must be:
- (a) right; clockwise.
  - (b) right; counter-clockwise.
  - (c) left; clockwise.
  - (d) left; counter-clockwise.
  - (e) two of the the above are correct.
8. If a proton is in a uniform magnetic field 37.3 mT (perpendicular to the proton's velocity), how many orbits will the proton have completed after 20.0  $\mu\text{s}$ ?
- (a) 12
  - (b) 75
  - (c) 22 thousand
  - (d) 137 thousand
  - (e) 3.6 million

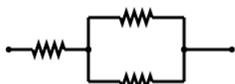
9. In a uniform electric field  $\vec{E} = (+32 \text{ N/C})\hat{i} + (-45 \text{ N/C})\hat{j}$  what is the difference in electric potential  $V_B - V_A$  if  $A$  is at  $x = -0.050 \text{ m}$ ,  $y = -0.200 \text{ m}$  and  $B$  is at  $x = -0.050 \text{ m}$ ,  $y = +0.200 \text{ m}$ ?
- $-22 \text{ V}$
  - $-18 \text{ V}$
  - $0 \text{ V}$
  - $+18 \text{ V}$
  - $+22 \text{ V}$
10. An electron, between the oppositely charged plates of a capacitor is launched towards the negative plate. As it moves closer to the negative plate
- $\Delta V > 0 \text{ V}$  and  $\Delta U_e > 0 \text{ J}$ .
  - $\Delta V > 0 \text{ V}$  and  $\Delta U_e < 0 \text{ J}$ .
  - $\Delta V < 0 \text{ V}$  and  $\Delta U_e > 0 \text{ J}$ .
  - $\Delta V < 0 \text{ V}$  and  $\Delta U_e < 0 \text{ J}$ .
11. What is the speed of an electron after it has crossed a potential difference of  $\Delta V = +9 \text{ V}$  if its initial speed was  $4.2 \times 10^6 \text{ m/s}$ ?
- $1.8 \times 10^6 \text{ m/s}$ .
  - $3.8 \times 10^6 \text{ m/s}$ .
  - $4.2 \times 10^6 \text{ m/s}$ .
  - $4.6 \times 10^6 \text{ m/s}$ .
12. We are given a solid sphere of conducting material that is charged.



Which pair of graphs are  $V(r)$  and  $E_r(r)$  for this charged object?

- Graph [1] is  $V(r)$  and graph [2] is  $E_r(r)$ .
- Graph [3] is  $V(r)$  and graph [2] is  $E_r(r)$ .
- Graph [3] is  $V(r)$  and graph [1] is  $E_r(r)$ .
- Graph [3] is  $V(r)$  and graph [4] is  $E_r(r)$ .
- Graph [4] is  $V(r)$  and graph [3] is  $E_r(r)$ .

13. Compare the equivalent resistance of these two groups of resistors:

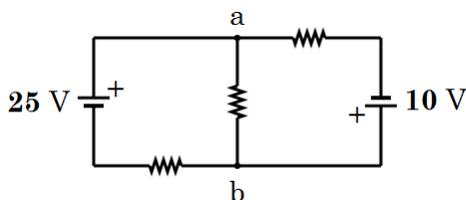


[1]



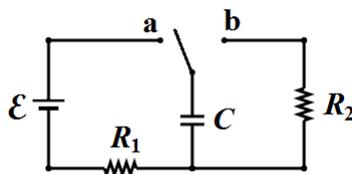
[2]

- (a)  $R_1 > R_2$ .  
 (b)  $R_1 = R_2$ .  
 (c)  $R_1 < R_2$ .
14. In the circuit shown below each resistor is  $100\ \Omega$ . The current flowing through the  $25\ \text{V}$  battery is  $200\ \text{mA}$ , and the current flowing through the  $10\ \text{V}$  battery is  $150\ \text{mA}$ .



The current through the resistor in the center is:

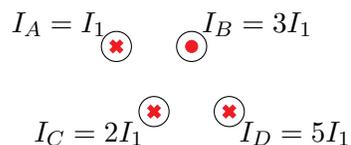
- (a)  $150\ \text{mA}$  from a towards b.  
 (b)  $150\ \text{mA}$  from b towards a.  
 (c)  $50\ \text{mA}$  from a towards b.  
 (d)  $50\ \text{mA}$  from b towards a.  
 (e)  $0\ \text{mA}$ .
15. Shown below is a charging/discharging circuit.



If  $R_1 > R_2$ , then which would take longer: charging, or discharging?

- (a) Charging.  
 (b) They would be the same because the capacitor is the same for both.  
 (c) Discharging.  
 (d) This can't be determined without knowing the value of the capacitance  $C$ .  
 (e) This can't be determined without knowing the value of the emf  $\mathcal{E}$ .

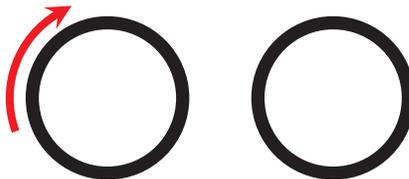
16. If an  $LC$  circuit oscillates with angular frequency  $\omega$ , by what factor must you change the inductance to make the new frequency  $\omega/2$ ?
- 4.
  - 2.
  - $\sqrt{2}$ .
  - 1/2.
  - 1/4.
17. A singly-ionized Helium atom has one electron, two protons, and two neutrons. The electric flux (measured in  $\text{N}\cdot\text{m}^2/\text{C}$ ) through a Gaussian surface that encloses this ion is:
- $-5.43 \times 10^{-8}$
  - $-1.81 \times 10^{-8}$
  - $+1.81 \times 10^{-8}$
  - $+5.43 \times 10^{-8}$
  - $+9.05 \times 10^{-8}$
18. Shown in cross-section are four long straight current-carrying wires that pass perpendicular to the page.



If a clockwise Amperian loop that encloses wire  $A$  only gives  $\oint \vec{B} \cdot d\vec{s} = +\mu_0 I_1$ , what Amperian loop would give  $\oint \vec{B} \cdot d\vec{s} = -4\mu_0 I_1$ ?

- a clockwise loop that encloses wires  $A$ ,  $B$  and  $C$  only.
- a counter-clockwise loop that encloses wires  $A$  and  $D$  only.
- a clockwise loop that encloses wires  $B$  and  $C$  only.
- a clockwise loop that encloses wires  $A$  and  $B$  only.
- a counter-clockwise loop that encloses wires  $B$ ,  $C$  and  $D$  only.

19. Two conducting loops are in the plane of the page, as shown.



- The current around the loop on the left is initially zero, increases up to a large clockwise current, then decreases back down to zero. The current induced in the loop on the right is:
- (a) initially clockwise, then counter-clockwise.
  - (b) initially clockwise, then clockwise again.
  - (c) initially counter-clockwise, then clockwise.
  - (d) initially counter-clockwise, then counter-clockwise again.
20. Current flowing through a 80 mH inductor increases from 400 mA to 1.2 A over 128  $\mu\text{s}$ . If we cross the inductor in the direction of current flow, the difference in electric potential is
- (a) +750 V
  - (b) +500 V
  - (c) -78 V
  - (d) -500 V
  - (e) -750 V

SPOILERS!

ANSWERS begin on the next page...

## Answers

### Problems

- (a)  $V = \int dV = \int_0^{2\pi} \frac{1}{4\pi\epsilon_0} \frac{\lambda R d\theta}{\sqrt{x^2 + R^2}} = \frac{\lambda 2\pi R}{4\pi\epsilon_0 \sqrt{x^2 + R^2}} = \frac{k_e Q}{\sqrt{x^2 + R^2}}$ .

(b) By symmetry  $E_y = E_z = 0$  N/C.  $E_x = -\frac{d}{dx} V(x) = \dots = \frac{k_e Q x}{(x^2 + R^2)^{3/2}}$ .
- (a) With a concentric sphere as the Gaussian surface between  $R_1$  and  $R_2$  the charge inside is  $Q_{\text{in}} = \rho(\frac{4\pi}{3}r^3 - \frac{4\pi}{3}R_1^3)$  (where we subtract the volume of the empty space to get the solid part). The flux integral is  $\oint \vec{E} \cdot d\vec{A} = E4\pi r^2$ . Gauss' Law gives  $E = k_e \frac{4\pi}{3} \rho(r - \frac{R_1^3}{r^2})$ .

(b) Outside the sphere the field is that of a point charge with  $Q = \rho \frac{4\pi}{3}(R_2^3 - R_1^3)$ , and the potential at the surface is  $V = k_e Q/R_2$ . Using the values given in the problem  $Q = -1.165 \times 10^{-13}$  C, and  $V = -3.49$  V.
- (a) The 2 A flowing from  $c$  to  $e$  splits evenly at  $e$ , with 1 A flowing through each 12  $\Omega$  resistor.

(b) Take the loop  $bacefdb$  to find  $+\mathcal{E}_1 - (12 \Omega)(1 \text{ A}) = 0$  V, which gives  $\mathcal{E}_1 = 12$  V.

(c) Choose  $I_2$  flowing from  $d$  to  $c$  and take the loop  $dcabd$  to find  $+\mathcal{E}_2 - (1 \Omega)I_2 - \mathcal{E}_1 = 0$  V, which gives  $I_2 = -8$  A (physically flowing from  $c$  to  $d$ ). Choose  $I_1$  flowing from  $b$  to  $a$  and take the junction  $c$  to find  $I_1 + I_2 - I_3 = 0$  A, which gives  $I_1 = +10$  A (physically flowing from  $b$  to  $a$ ).

(d) Since current is flowing from  $+$  to  $-$  across the battery, it is absorbing energy at a rate  $P = |I_2 \mathcal{E}_2| = 32$  W.
- (a) When the capacitor is uncharged current flows across it as if it were a wire of no resistance. In the circuit this is equivalent to joining junctions  $c$  and  $d$  into a single junction. The 1 $\Omega$  and 8 $\Omega$  resistors in parallel ( $R_{\text{eq}} = \frac{8}{9}\Omega$ ) are then in series with the 4 $\Omega$  and 2 $\Omega$  resistors in parallel ( $R_{\text{eq}} = \frac{4}{3}\Omega$ ). The equivalent resistance of the whole circuit, at that moment, is  $R_{\text{eq}} = \frac{20}{9}\Omega$ , and the current flowing through the battery is  $\frac{9}{2}$  A. This means that the voltage across the 1 $\Omega$  resistor (as well as the 8 $\Omega$ ) is  $(\frac{9}{2} \text{ A})(\frac{8}{9}\Omega) = 4$  V. This also means that the voltage across the 4 $\Omega$  resistor (as well as the 2 $\Omega$ ) is 6 V. The current through the 1 $\Omega$  is  $4 \text{ V}/1\Omega = 4$  A and the current through the 4 $\Omega$  is  $6 \text{ V}/4\Omega = \frac{3}{2}$  A. Consequently a current of  $\frac{5}{2}$  A is flowing into the capacitor at this moment.

(b) After a very long time the capacitor will be fully charged, with no current flowing across it. It behaves like an open gap in the circuit. Current flows in the branch  $acb$  separately from the branch  $adb$ . Solving for these currents we get  $I_{acb} = (10 \text{ V})/(1\Omega + 4\Omega) = 2$  A and  $I_{adb} = (10 \text{ V})/(8\Omega + 2\Omega) = 1$  A. The voltage difference across the 1 $\Omega$  resistor is  $1\Omega \cdot 2 \text{ A} = 2$  V, and the voltage difference across the 8 $\Omega$  resistor is  $8\Omega \cdot 1 \text{ A} = 8$  V. This gives the voltage difference across the capacitor as 6 V (with the point  $c$  at the higher potential). Thus the charge on the capacitor is  $Q = C \Delta V = (1 \mu\text{F})(6 \text{ V}) = 6 \mu\text{C}$ .

(c) With the battery disconnected the capacitor will discharge through branch  $cad$  in parallel with branch  $abd$ . The equivalent resistance that the capacitor faces is  $[(1\Omega + 8\Omega)^{-1} + (4\Omega + 2\Omega)^{-1}]^{-1} = \frac{18}{5} \Omega = 3.6 \Omega$ . The time constant for the discharge is thus  $RC = (3.6 \Omega)(1 \mu\text{F}) = 3.6 \mu\text{s}$ . Discharging to one-tenth of its initial value means  $e^{-t/RC} = \frac{1}{10}$ , which gives  $t = -RC \ln(\frac{1}{10}) = 8.29$  s.

5. (a) The distance from wire 1 to the point  $P$  is  $r = 0.374$  m. The magnitude of the magnetic field due to wire 1 by itself is  $\mu_0 I / 2\pi r = 1.709 \mu\text{T}$ . Draw a line from wire 1 to the point  $P$ . This magnetic field contribution is perpendicular to that, pointed up (the  $+y$ -direction) and slightly to the right (the  $+x$ -direction). Consequently the  $x$ -component of this is  $+0.607 \mu\text{T}$ . (Draw the diagram to see the correct triangles!) The contribution due to wire 2 will have the same magnitude and same  $x$ -component, but will have the opposite  $y$ -component (which will cancel!). Neither wire contributes to the  $z$ -component. The net magnetic field is thus  $B_x = +1.214 \mu\text{T}$ ,  $B_y = 0$  T and  $B_z = 0$  T (alternatively  $\vec{B} = (+1.214 \mu\text{T})\hat{i}$ ).
- (b) Since  $q = -1.602 \times 10^{-19}$  C and  $\vec{v} = (+5200 \text{ m/s})\hat{k}$ , the magnetic force is  $\vec{F}_m = q\vec{v} \times \vec{B} = (-1.012 \times 10^{-21} \text{ N})\hat{j}$  (pointing in the  $-y$ -direction).
- (c) Requiring  $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = q\vec{E} + \vec{F}_m = \vec{0}$  gives  $\vec{E} = -\vec{F}_m/q = (-6.314 \times 10^{-3} \text{ N/C})\hat{j}$ . (Careful! There's three minus signs inside that expression.) Thus  $E_x = 0$  N/C,  $E_y = -0.006314$  N/C and  $E_z = 0$  N/C.
6. (a) The magnetic field is out of the page. The magnet flux through the coil points out of the page, and is increasing. The induced, opposing flux will have to point into the page. The induced current must hence circulate clockwise around the coil. The rate of change of flux through the loop is  $d\Phi_m/dt = BLv = (2.00 \text{ T})(0.200 \text{ m})(0.500 \text{ m/s}) = 0.200 \text{ V}$ . The induced emf around the coil is  $|\mathcal{E}| = N d\Phi_m/dt = 2.00 \text{ V}$ . Thus  $I = \mathcal{E}/R = (2.00 \text{ V})/(50.0 \text{ ohm}) = 40.0 \text{ mA}$ .
- (b) The forces on the top and bottom edges cancel, leaving the force on the right edge only:  $F = ILB = (0.0400 \text{ A})(0.200 \text{ m})(2.00 \text{ T}) = 16.0 \text{ mN}$ . That is the force on one wire – but it wraps 10 times around the loop, and each receives the same force. So the net magnetic force is ten times that:  $F = 0.160 \text{ N}$ . The current flows downwards on the page and the field is out of the page, so the force points to the left on the page.
- (c) Since the coil moves with constant speed (we are told), the force we apply must balance the oppositional force applied by the magnetic field. The power we supply must thus be  $P = \vec{F} \cdot \vec{v} = Fv = (0.160 \text{ N})(0.500 \text{ m/s}) = 0.080 \text{ W}$ . This process lasts  $\Delta t = \Delta x/v = (0.400 \text{ m})/(0.500 \text{ m/s}) = 0.800 \text{ s}$ , and the work we do is  $W = P \Delta t = (0.080 \text{ W})(0.800 \text{ s}) = 0.064 \text{ J}$ . By conservation of energy, this work we do must equal the energy dissipated by the resistor. (Alternatively, we could note that  $P = I^2 R = (0.040 \text{ A})^2(50.0 \text{ ohm}) = 0.080 \text{ W}$ , and proceed similarly.)

### Multiple Choice

- |        |        |         |         |         |
|--------|--------|---------|---------|---------|
| 1. (c) | 5. (b) | 9. (d)  | 13. (a) | 17. (c) |
| 2. (c) | 6. (d) | 10. (c) | 14. (c) | 18. (e) |
| 3. (d) | 7. (e) | 11. (d) | 15. (a) | 19. (a) |
| 4. (e) | 8. (a) | 12. (a) | 16. (a) | 20. (d) |