Dawson College

Mathematics Department

Final Examination

201-NYB-05, Calculus II- Regular   Section 03

Thursday, December 17, 2015   9:30-12:30

Student Name: _____**Student I.D. #:**_____

Instructor: O.Veres

INSTRUCTIONS:

- Print your name and student number in the space provided above.
- Attempt all questions. Show all your work.
- All questions are to be answered directly on the examination paper.
- Only the following calculators are permitted:
  EL-531 XG, EL-531 X
- Translation and regular dictionaries are permitted.
- This examination consists of 14 questions on 19 pages, including this cover page.
- Please ensure that you have a complete exam package before starting.
- The examination must be returned intact.

1. [4 marks] Using the limit of a Riemann Sum, evaluate:

$$\int_0^4 (6x^2 - 5)dx$$

NOTE: $\sum_{i=1}^{n} c = cn,$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2},$ $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

2. [25 marks] Find the following integrals:
   a. $\int \sec^4(3x)\tan(3x)\, dx$

   b. $\int x^2 \ln x\, dx$

c. $\int \frac{\sqrt{x}dx}{x+\sqrt{x}}$

d. $\int \frac{x^2}{\sqrt{1-x^2}} dx$

e. $\int \frac{x^2-x-7}{(x-2)(x^2+1)} dx$

3. [4 marks] Find the average value of $f(x) = x\sqrt{x^2 + 36}$ over the interval $[0, 8]$

4. [8 marks] Evaluate the following limits

$$a.)\qquad \lim_{x \to 0} \frac{2x^2 - 3x + \sin 3x}{e^{2x} + 2e^x - 4x - 3}$$

b.) $\lim\limits_{x \to 0}(1 + 7x)^{2/x}$

5. [8 marks] Evaluate the improper integral if it converges or show that it diverges

$$a.) \quad \int_{2}^{4} \frac{1}{(x-2)^2} dx$$

b.)　$\int_{-\infty}^{0} \frac{xe^{-x^2}}{1+e^{-x^2}} dx$

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6. [5 marks]   Find the length of the curve $y = \frac{1}{8}x^2 - \ln x$ over the interval $[2,6]$

7. [5 marks] Find the area of the region bounded by the graphs of $y = x^2 + 2, y = -x^2 + 4$

8. [8 marks]   Find the volume of the solid generated when the region bounded by

$$y = \frac{x}{2} \qquad \text{and} \qquad y = \sqrt{x} \qquad \text{is rotated about}$$

    a.   the x- axis

    b. the y-axis

9. [5 marks]   Find

$$\frac{d}{dx}\left(\int_x^{e^x} \csc(t^2 + 1)dt\right)$$

10. [4 marks] A population of whales grows at a rate $\frac{dP}{dt} = \frac{2}{3}P^{1/4}$, where t is the time in years. How many whales will there be in 38 years if there were 16 when $t = 0$.

11. [4 marks]   Find the sum of the infinite series

$$\sum_{n=1}^{\infty} \left(\frac{3^n}{4^{n-1}} + \frac{1}{5^n} \right)$$

12. [12 marks] Test the following series for convergence or divergence using an appropriate test.

Clearly state the test used.

$$a.) \quad \sum_{n=1}^{\infty} \frac{n^3 + 2n^2 - 1}{2 + n + 4n^3}$$

b.) $\sum_{n=1}^{\infty} \dfrac{(-1)^n 3^{n+1}}{(n-1)!}$

c.) $\displaystyle\sum_{n=1}^{\infty} \dfrac{n}{n^3 + 2n^2 - 1}$

13. [4 marks] Find the radius of convergence and the open interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(n+1)(x+2)^n}{4^n}$$

14. [4 marks] Find the 3rd Taylor polynomial of the function $f(x) = \dfrac{1}{3x+2}$ at $x = 1$.

ANSWERS:

1. 108
2.
    a. $\frac{1}{12}\sec^4(3x) + C$ or $\frac{1}{12}\tan^4(3x) + \frac{1}{6}\tan^2(3x) + C$

    b. $\frac{1}{9}x^3(3\ln x - 1) + C$

    c. $2\sqrt{x} - 2\ln\left|\sqrt{x} + 1\right| + C$ or $2(\sqrt{x} + 1) - 2\ln\left|\sqrt{x} + 1\right| + C$

    d. $\frac{1}{2}\arcsin x - \frac{x\sqrt{1-x^2}}{2} + C$

    e. $-\ln|x - 2| + \ln(x^2 + 1) + 3\arctan x + C$

3. $\frac{98}{3}$
4. a. $\frac{2}{3}$ b. e^{14}
5. a. ∞ Diverges b. $-\frac{1}{2}\ln 2$ Converges
6. $4 + \ln 3$
7. $\frac{8}{3}u^2$
8. a. $\frac{8}{3}\pi u^3$ b. $\frac{64}{15}\pi u^3$
9. $-\csc(x^2 + 1) + e^x\csc(e^{2x} + 1)$
10. 81
11. $\frac{49}{4}$
12.   a. diverges, by Test for Divergence, b. converges, by Ratio Test, c. converges, by Comparison or Limit Comparison Test
13. $R = 4, (-6,2)$
14. $\frac{1}{5} - \frac{3}{25}(x - 1) + \frac{9}{125}(x - 2)^2 - \frac{27}{625}(x - 1)^3$