Dawson College

Mathematics Department

Final Examination

201-NYA-05- Calculus I Section: 03

LABORATORY TECHNOLOGY-ANALYTICAL CHEMISTRY PROGRAM

Monday, May 16, 2016 14:00-17:00

Student Name: _____Student I.D. #:_____

Instructor: O. Veres

INSTRUCTIONS:

- Print your name and student number in the space provided above.
- Attempt all questions. Show all your work.
- All questions are to be answered directly on the examination paper.
- Only the following calculators are permitted: EL-531 XG or EL-531X
- Translation and regular dictionaries are permitted.
- This examination consists of 15 questions on 17 pages, including this cover page.
- Please ensure that you have a complete exam package before starting.
- The examination must be returned intact.

Question#	Marks obtained/available
1	/12
2	/5
3	/5
4	/14
5	/4
6	/4
7	/5
8	/5
9	/5
10	/4
11	/8
12	/4
13	/5
14	/5
15	/15
Total/100	

1. [12 marks] Evaluate the given limits

$a.\ \lim_{x\to 2^-} \dfrac{x^2-4}{x^2-4x+4}$

$b.\ \lim_{x\to 5} \dfrac{\sqrt{x+4}-3}{10-2x}$

$c.\ \lim_{x\to -\infty} \dfrac{2-4x^3}{41+x^2}$

2. [5 marks] Determine whether the function is continuous on its entire domain. Justify your answer.

$$f(x) = \begin{cases} 2\cos 3x & if \quad x < 0 \\ 2^{1-x^3} & if \quad x = 0 \\ \sqrt{5x+4} & if \quad x > 0 \end{cases}$$

3. [5 marks] Use the definition of derivative to compute the derivative of the given function.

$$f(x) = \frac{3}{x-2}$$

4. [8+6 marks]
   a. Differentiate each function. **Do not** simplify your answer.

      $i.\quad y = (\sin x)^{\cos x}$

ii. $f(\theta) = e^{\cos(e^{\theta})} \arcsin\sqrt{3\theta}$

b. Find the indefinite integral

i. $\int \left(\frac{2}{x^2+1} + \frac{\sqrt[3]{x}+x}{x^2} \right) dx$

ii. $\int \left(\frac{1 + \sin^3 x}{\sin^2 x} \right) dx$

5. [4 marks] The amount n (in g) of a compound formed during a chemical reaction is given by $n = \frac{5t^2}{3t^2+9}$; where t is the time in $min.$ Find the rate of change of the compound when $t = 2.00\ min.$

6. [4 marks] A metal sphere dissolves in acid such that its volume decreases by $0.25mm^3/min$. How fast is the radius of the sphere changing when the volume is $36\pi\ mm^3$?

7. [5 marks] A right circular cylinder is inscribed in a cone with height of $12\ cm$ and base radius of $4\ cm$. Find the largest possible volume of such a cylinder.

8. [5 marks] Find the normal line to the curve $\sin(xy) + 2y\cos x = 6$ at $(0,3)$.

9. [5 marks] Find the value(s) of x for which the tangent line to the curve $y = (x^2 + 3)^3(2x - 1)^2$ is horizontal.

10. [4 marks] Evaluate

$$\lim_{n\to\infty}\left(\frac{81}{n}\sum_{i=1}^{n}\sqrt{\frac{81i}{n}}\right)$$

11. [4+3+1 marks] Evaluate the given definite integral

$$\int_0^3 (2x-4)dx$$

a. using the Limit of Riemann Sums : Formulae: $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$; $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$;

b. using geometry

c. using the Fundamental Theorem of Calculus Part 2.

12. [4 marks] Given $\int_2^0 f(s)ds = 3$; $\int_2^7 f(s)ds = -5$; $\int_0^7 g(s)ds = 6$; $\int_0^2 g(s)ds = 4$, find

$$\int_0^2 3f(s)ds - \int_2^7 \big(f(s) - 2g(s)\big)ds =$$

13. [5 marks] Find the total area enclosed by the functions $f(x) = x^3 - 2x + 4$ and $g(x) = 2x + 4$

14. [5 marks] Find the average value of the function $f(x) = 3\sin x - \cos x$ on $[0, \pi/2]$

15. [1+2+3+2+1+2+1+3 marks] Given $y = \frac{x^2-1}{(x-2)^2}$ $y' = \frac{2(1-2x)}{(x-2)^3}$ $y'' = \frac{2(4x+1)}{(x-2)^4}$

a. Find the domain of the function

b. Find the x- intercepts and y -intercept

c. Find the horizontal and vertical asymptotes

d. Find the intervals where the function is increasing and decreasing (a table is given on page 17)

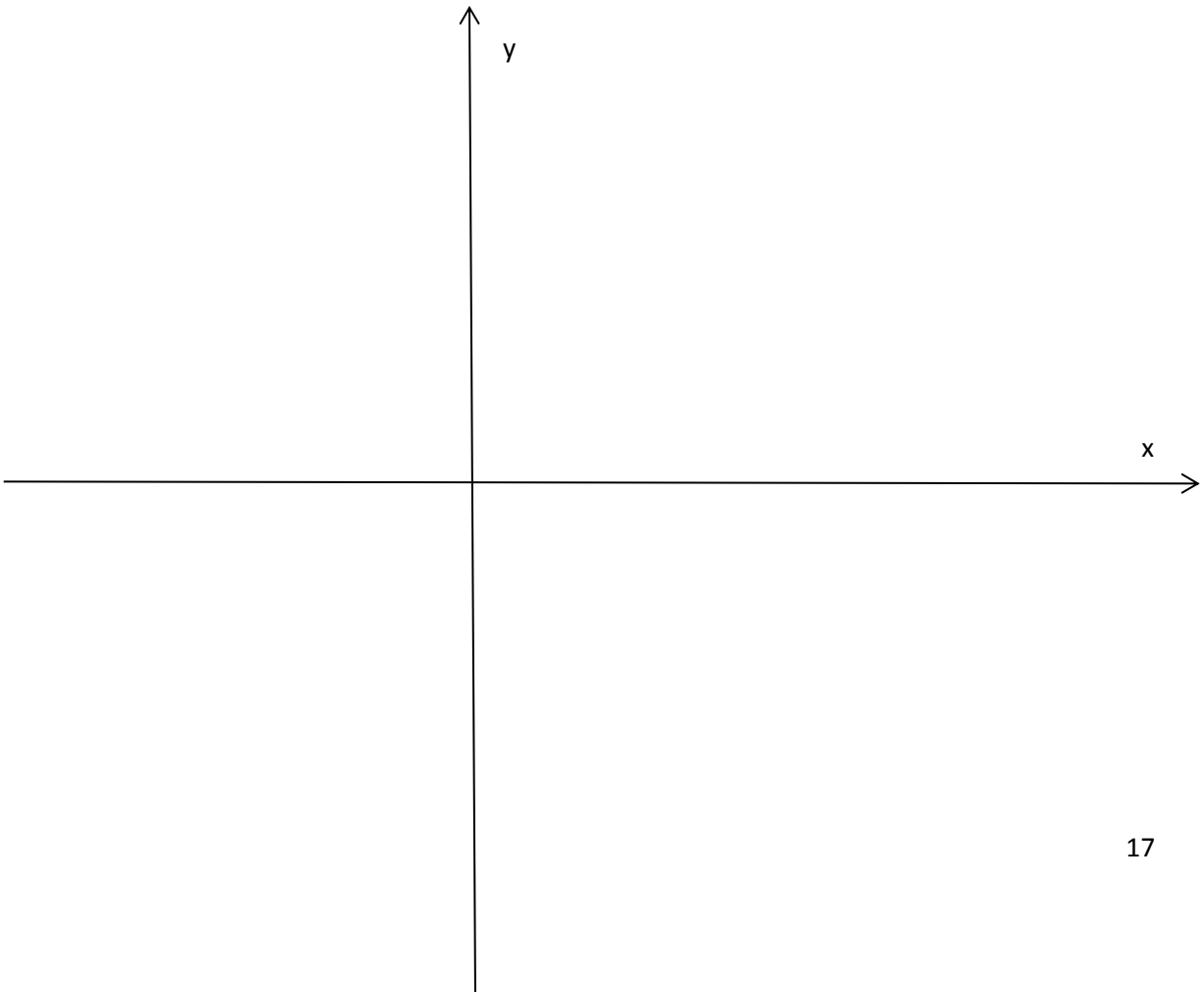
e. Find the local minimum and maximum values of the function

f. Find the intervals of concavity

g. Find the inflection points

h. Using the above information sketch the graph of this function labeling the points found in the previous steps.

x	
f'(x)	
f"(x)	
f(x)	
G_f	



ANSWERS:

1. $a.\ -\infty,\quad b.\ -\frac{1}{12}\quad c.\infty$
2. The function is continuous on its entire domain.
3. $f'(x) = -\frac{3}{(x-2)^2}$
4. $a.\ \ i.\, y' = (\sin x)^{\cos x}\left(-\sin x \ln(\sin x) + \frac{\cos^2 x}{\sin x}\right);$

   $ii.\ f'(\theta) = -\sin\left(e^{\theta}\right) e^{\cos(e^{\theta})} e^{\theta} \arcsin\sqrt{3\theta} + e^{\cos(e^{\theta})} \frac{1}{\sqrt{1-3\theta}} \frac{3}{2\sqrt{3\theta}}$

   $b.\ i.\quad 2\arctan x - \frac{3}{2}\frac{1}{\sqrt[3]{x^2}} + \ln|x| + C;\quad ii. -\cot x - \cos x + C$
5. $0.408\ g/min$
6. The radius is decreasing at a rate of $0.0022\ mm/min$
7. $V = \frac{256\pi}{9} cm^3$
8. $y = \frac{2}{3}x + 3$
9. $x = \frac{1}{2}$
10. 486
11. a, b, c : -3
12. 0
13. $8u^2$
14. $\frac{4}{\pi}$
15. $a.\ (-\infty, 2) \cup (2, \infty);\qquad b.\ \left(0, -\frac{1}{4}\right); (-1,0);\ (1,0);\qquad c.\qquad y = 1\ \ is\ \ HA\ \ to\ \infty;$

    $x = 2\quad VA\ to + \infty;\qquad d.\quad f\ decreases\ on\ \left(-\infty, \frac{1}{2}\right) \cup (2, \infty);\quad f\ increases\ on\ \left(\frac{1}{2}, 2\right);$

    $e.\quad f\left(\frac{1}{2}\right) = -\frac{1}{3}\ relative\ minimum;$

    $f.\quad the\ graph\ of\ the\ function\ is\ \ concave\ upward\ on\ \left(-\frac{1}{4}, 2\right) \cup (2, \infty)$

    $and\ \ it\ is\ concave\ downward\ \ on\ \left(-\infty, -\frac{1}{4}\right);$

    $g.\left(-\frac{1}{4}, -\frac{15}{81}\right) inflection\ point$