# DAWSON COLLEGE
# Mathematics Department
# Final Examination
# Calculus I
# 201-103-DW (Commerce & IBS)
# May 19th, 2015

**Student Name** ______________________________________________

**Student I.D. #** ______________________________________________

**Teacher** ____________________________________________________

**Instructors**: Gorelyshev, I., Farnesi, C., Jimenez, A.
TIME: 09:30 – 12:30

| Question | Marks |
| --- | --- |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |
| Total / | |

**Instructions:**

- Print your name and student ID number in the space provided above.
- All questions are to be answered directly on the examination paper in the provided space.
- Translation and regular dictionaries are permitted.
- Small, noiseless, non-programmable calculators without text storage or graphic capability are permitted.
- **No formula sheet is provided.**

- This examination consists of 13 questions.
- Apart from the attestation page (preceding this page), there are 14 pages including this page. .
- **This exam booklet must be returned intact.**

[12 Marks]

1) Evaluate the following limits. Write “dne” when the limit doesn't exist:

a) $\lim\limits_{x\to 3} \dfrac{\sqrt{2x+3}-3}{x-3}$ [4 Marks]

$$\frac{\sqrt{2\cdot 3+3}-3}{3-3} = \frac{\sqrt{9}-3}{3-3} = \frac{0}{0} - \text{case}$$

rationalizing the expression $\dfrac{\sqrt{2x+3}-3}{x-3} \cdot \dfrac{\sqrt{2x+3}+3}{\sqrt{2x+3}+3}$

$$= \frac{\overbrace{2x+3-9}^{2x-6}}{(x-3)(\sqrt{2x+3}+3)} = \frac{2\cancel{(x-3)}}{\cancel{(x-3)}(\sqrt{2x+3}+3)}$$

$$\lim_{x\to 3} \frac{2}{\sqrt{2x+3}+3} = \frac{2}{\sqrt{2\cdot 3+3}+3} = \frac{2}{6} = \frac{1}{3}$$

b) $\lim\limits_{x\to 0} \dfrac{x-4}{4x-x^2}$ [4 Marks]

$$\frac{0-4}{4\cdot 0-0^2} = \frac{-4}{0} - \text{case} \qquad \frac{x-4}{x(4-x)} = -\frac{1}{x}$$

therefore $\lim\limits_{x\to 0} \dfrac{x-4}{4x-x^2}$ (d.n.e.) or $(-\infty)$

decreases without bounds as $x \to 0$

c) $\lim_{x \to \infty} \dfrac{x^3 - x + 3}{x^4 - 2x^2 - 2} = 0$ [4 Marks]

$3 = \deg(N) < \deg(D) = 4$

[5 Marks]

2) Use the definition of continuity to determine whether $f(x)$ is continuous at $x = -2$

$$f(x) = \begin{cases} \dfrac{2x-3}{x+2} & \text{if} \quad x < -2 \\ -3 & \text{if} \quad x = -2 \\ 2x+1 & \text{if} \quad x > -2 \end{cases}$$

$-3 = f(-2) \neq \lim_{x \to -2} f(x)$ dne

$\Rightarrow f(x)$ is discontinuous at $x = -2$

[5 Marks]

3) Use the limit definition of the derivative to find the derivative of

$$f(x)=x^2-x+7$$

Note: Do not use any of the differentiation rules here.

$$f'(x) = \lim_{t\to x} \frac{f(t)-f(x)}{t-x} =$$

$$\lim_{t\to x} \frac{t^2-x^2-t+x}{t-x} =$$

$$\lim_{t\to x} \frac{\cancel{(t-x)}(t+x-1)}{\cancel{t-x}} = 2x-1$$

[16 Marks]

4) Find y ′ (**<u>Do not simplify</u>**):

a) $y=\ln\left(x^3\sqrt{x^2+1}\right)$ [4 Marks]

$$\left[3\ln x + \frac{1}{2}\ln(x^2+1)\right]' =$$

$$= \frac{3}{x} + \frac{1}{2}\,\frac{2x}{x^2+1}$$

b) $y = \cos(x^3)\tan^3(x)$ [4 Marks]

$$y' = [\cos(x^3)]'\tan^3(x) + \cos(x^3)[\tan^3(x)]'$$

$$= -\sin(x^3)3x^2\tan^3(x) + \cos(x^3)3\tan^2(x)\sec^2(x)$$

c) $y = \dfrac{e^{\cos(x)}}{2x+3}$ [4 Marks]

$$y' = \frac{[e^{\cos(x)}]'(2x+3) - e^{\cos(x)}[2x+3]'}{(2x+3)^2}$$

$$= \frac{-\sin(x)e^{\cos(x)}(2x+3) - e^{\cos(x)}\cdot 2}{(2x+3)^2}$$

d) $y=e^{\sin(x^2)}$ [4 Marks]

$y' = \cos(x^2)\,2x\,e^{\sin(x^2)}$

[5 Marks]

5) Use logarithmic differentiation to find $y'$ given that $y=(x+1)^{x^2}$

$y' = (x+1)^{x^2}\left[x^2 \ln(x+1)\right]'$

$= (x+1)^{x^2}\left(2x \ln(x+1) + \dfrac{x^2}{x+1}\right)$

[5 Marks]

6) Use implicit differentiation to find $\frac{dy}{dx}$ (or y') at (1,2) if $x^2+xy+y^2=7$

$$2x+y+xy'+2yy'=0$$

$$y'=\frac{-2x-y}{x+2y}$$

$$y'(1,2)=\frac{-2\cdot1-2}{1+2\cdot2}=\frac{-4}{5}$$

[10 Marks]

7) Given the following function

$$f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-2x+6$$

a) Find the values of x where the tangent to function f(x) is horizontal. [5 Marks]

$$f'(x)=x^2+x-2=(x+2)(x-1)=0$$

$$\Rightarrow x_1=-2 \quad x_2=1$$

b) Find the absolute extrema of the given function over the closed interval $[0,2]$

[5 Marks]

$f(0) = 6$

$f(1) = 4 + \frac{5}{6}$ min

$f(2) = 4 + \frac{8}{3} = 6 + \frac{2}{3}$ Max

[10 Marks]

8) Given function y(x) below

$$y(x) = x^2 \ln(x) + 5$$

a) Write the equation of the tangent to y(x) at (1,5). [5 Marks]

$y'(x) = 2x \ln x + x$

$m = y'(1) = 1$

$5 = 1 + b \Rightarrow b = 4$

$\boxed{y = x + 4}$

b) Calculate y"(1). [5 Marks]

$y'' = 2\ln x + 2 + 1$

$y''(1) = 3$

$\frac{4}{3} - 4 - 12 - 4 = -20 + \frac{4}{3}$

$1\frac{1}{3}$

[14 Marks]

9) Given $f(x) = \frac{4}{3}x^3 - 4x^2 - 12x - 4$

a) Find the intervals in which $f(x)$ is increasing, decreasing. [3 Marks]

$f'(x) = 4x^2 - 8x - 12 = 4(x^2 - 2x - 3)$

$= 4(x-3)(x+1) = 0$

$x_1 = -1 \qquad x_2 = 3$

$f'(-2) > 0$ increasing

$f'(0) < 0$ decreasing

$f'(4) > 0$ increasing

$(-\infty, -1) \cup (3, \infty)$ increasing

$(-1, 3)$ decreasing

b) Find all the maximum and minimum points. [3 Marks]

Max $(-1, f(-1)) = (-1, 2\frac{2}{3})$

min $(3, f(3)) = (3, -40)$

c) Find the intervals in which $f(x)$ is concave up and concave down. [3 Marks]

$f''(x) = 8x - 8 = 8(x-1) = 0$

$x = 1$

$f''(0) < 0$ $\downarrow$ downward

$f''(2) > 0$ $\uparrow$ upward

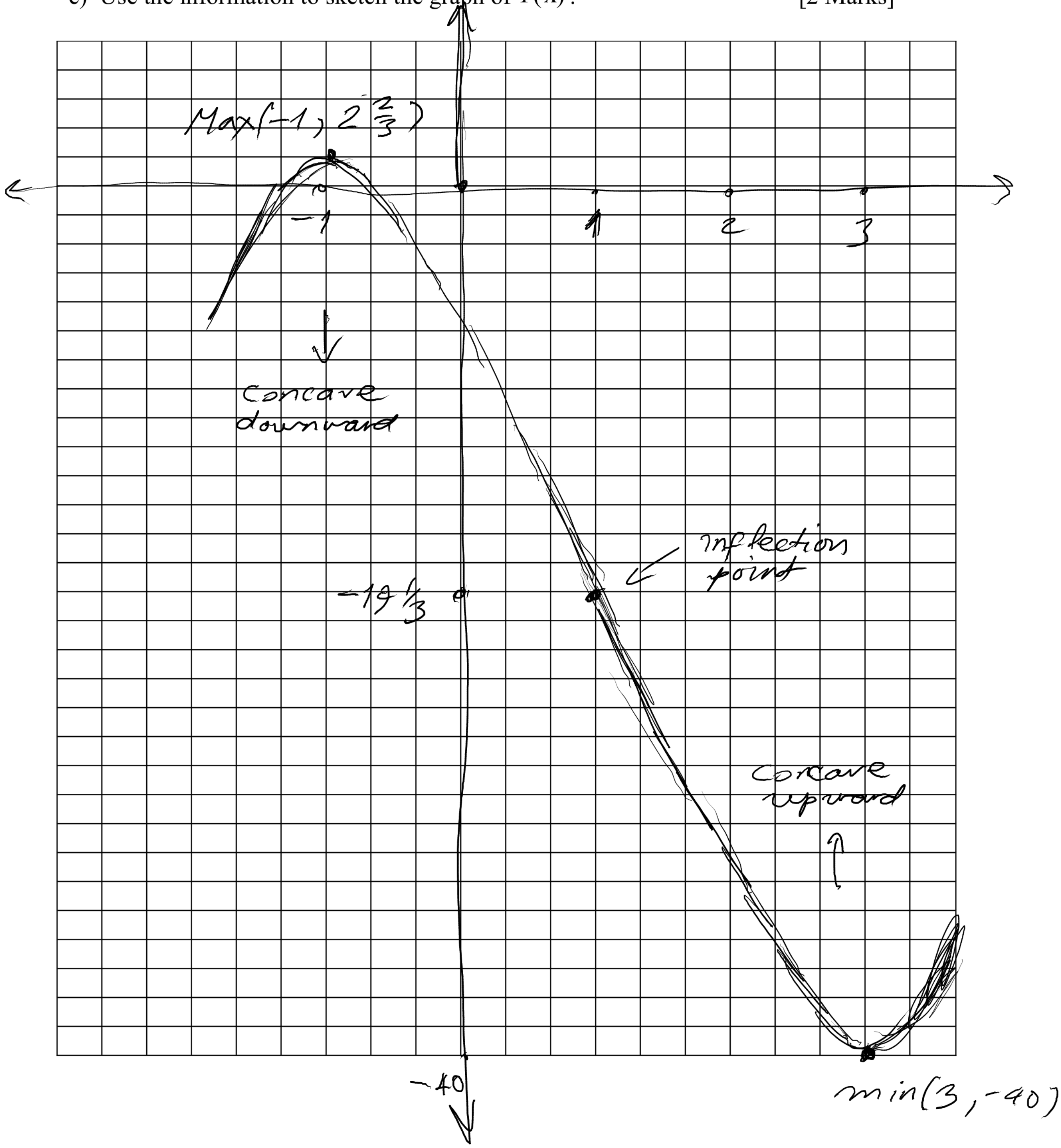
$(-\infty, 1)$ concave downward

$(1, \infty)$ concave upward

d) Find the points of inflection, if any. [3 Marks]

$(1, f(1)) = (1, -19\frac{1}{3})$

e) Use the information to sketch the graph of $f(x)$. [2 Marks]



[5 Marks]

10) The weekly demand and cost functions for $x$ units of a Samsung-tablet model are given below:

$p(x) = -0.06x + 180$ : weekly demand in \$/item $(0 \le x \le 3000)$ and

$C(x) = 0.0002x^3 - 0.02x^2 + 12x + 600$ : weekly cost function in \$ for x units .

a) Use marginal profit to estimate the profit realized from selling the 201st unit. [3 Marks]

$P(x) = R(x) - C(x) =$

$-0.06x^2 + 180x - 0.0002x^3 + 0.02x^2 - 12x - 600$

$P'(x) = -0.0006x^2 - 0.08x + 168$

$P'(200) = 168 - 40 = 128$

b) Find the production level x that gives the maximum profit. [2 Marks]

$P'(x) = 0$ solve for $x$

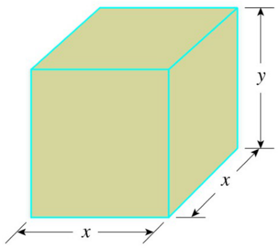
$-0.0006x^2 - 0.08x + 168 = 0$

$6x^2 + 800x - 1680000 = 0$

$x = 5600/12 \approx 467$

[5 Marks]

11) A rectangular box with a square base, like the one shown below, has a total capacity of 20 $cm^3$. If the material for the top costs \$2 per $cm^2$, the material for the sides costs \$10 per $cm^2$ and the material for the base costs \$3 per $cm^2$, determine the dimensions x and y of the box that can be constructed at minimum cost.




$$C(x,y) = 2x^2 + 10 \cdot 4xy + 3x^2 = 5x^2 + 40xy$$

$$20 = x^2 y \Rightarrow \boxed{y = \frac{20}{x^2}}$$

$$C(x) = 5x^2 + 40x \cdot \frac{20}{x^2} = 5x^2 + \frac{40 \cdot 20}{x}$$

$$C'(x) = 10x - \frac{40 \cdot 20}{x^2} = 0$$

$$x^3 = \frac{40 \cdot 20}{10}$$

$$\boxed{x = 2\sqrt[3]{100}}$$

$$\boxed{y = \frac{20}{4\sqrt[3]{100}^2}}$$

[4 Marks]

12) Find $\int \frac{3x^4+3x-1}{x^3}\,dx$ .

$$\int 3x + \frac{3}{x^2} - \frac{1}{x^3}\,dx$$

$$= \frac{3x^2}{2} - \frac{3}{x} + \frac{1}{2x^2} + C$$

[4 Marks]

13) Find $\int x(2x^2-4)^6\,dx$ .

$$\int x u^6 \frac{1}{4x}\,du = \frac{1}{4}\int u^6\,du =$$

$$\frac{u^7}{4\cdot 7} = \frac{(2x^2-4)^7}{28} + C$$