DAWSON COLLEGE

MATHEMATICS DEPARTMENT

Final Examination

Mathematics 201-401-DW
Statistics for Social Science
(Section 01)
Instructor: Mélanie Beck

Date: Tuesday, December 19, 2017
Time: 9:30 - 12:30

1. ***(6 marks)*** Find the probability that when two cards are drawn from a standard deck of 52 without replacement, the second card drawn is a heart. (You can use a tree diagram if you want to.)

2. ***(3 marks)*** The organizer of a contest must form a committee consisting of six persons. He has been suggested 5 names of artists, 3 of politicians and 6 of professors. How many different committees can be formed?

3. ***(8 marks)*** Employment data at a large company reveal that 72% of the workers are married, 44% of the workers are university graduates, and half of the university graduates are married. What is the probability that a randomly chosen worker is:

   (a) married and a university graduate?

   (b) married, given that he/she is a university graduate?

   (c) married or a university graduate?

   (d) neither married, nor a university graduate?

4. ***(5 marks)*** A dishwasher seller established, using previous years sales, that the number x of dishwashers sold daily follows the following probability distribution:

Number of dishwashers sold daily	Probability
0	0.18
1	0.35
2	0.26
3	0.21

   (a) Verify that this is a probability distribution.
   (b) Compute the number of dishwashers he can expect to sell during a day.
   (c) Compute the standard deviation of this probability distribution.

5. ***(8 marks)*** According to a U.S. Government study, 68% of children live in homes with two married parents. Assuming that this result hold true for all children currently, find the probability that in a random sample of 12 children, the number who live in homes with two married parents is
   (a) exactly 5;
   (b) at most 2.
   (c) What is the expected value and the standard deviation of this distribution?

6. ***(4 marks)*** Let Z be a random variable with a standard normal distribution.
   (a) Find $P(-1.1 < z < -0.72)$
   (b) Draw the standard normal curve and shade the area corresponding to (a).

7. ***(6 marks)*** Let X be a continuous random variable that is normally distributed with a mean of 40 and a standard deviation of 4. Find the probability that X assumes a value
a. between 29 and 36
b. less than 40
c. greater than 32

8. ***(6 marks)*** Suppose a college claims that its graduates from the business program earn at least $40,000 a year, on average, in the first year after graduation. Assume that the year-after-graduation salaries are normally distributed, with a standard deviation of $1,500. You survey a random sample of 20 graduates of the program, and find that, one year after graduation, the average salary is $38,000.
(a) What would the sampling distribution of the $\bar{x}$-values be?
(b) What is the probability of getting an $\bar{x}$-value as low as $38,000 (that is $38,000 or lower)?
(c) Does this mean that the average salary of graduates from the business program is less than $40,000 in the first year after graduation?

9. ***(7 marks)*** In a survey conducted in April 2004, 43% of chief security officers (CSOs) said that electronic crime had increased at their companies during the previous year. Assume that this percentage is true for the current population of CSOs. Find the probability that in a random sample of 400 CSOs, the number who will hold the above mention opinion is 160 to 180.

10. ***(5 marks)*** A researcher wants to estimate the average grocery bill of households whose members shop at a particular supermarket to within 10$, with 95% confidence. How big a sample size is necessary? The standard deviation is known to be $32.45.

11. ***(6.5 marks)*** A college is concerned that the housing boom in the immediate area has made it difficult for students to find affordable accommodation. It is claimed that the average monthly rent paid by a student at the college is $500. A random sample of 40 students are surveyed, and their monthly rent costs are recorded. The sample mean was $543.21, with a sample standard deviation of $47.89. Construct and interpret a 95% confidence interval estimate for the average monthly rent paid by students at this college.

12. ***(6.5 marks)*** The manager of a research institute is thinking of subsidizing the professional membership fees for the institute's scientists. Before proceeding, the manager decides to estimate the proportion of scientists who belong to professional associations, and finds that 40% of a random sample of 75 scientists belong to professional associations. Construct and interpret a 90% confidence interval for the proportion of all scientists who belong to professional associations.

13. ***(8 marks)*** The director of a state agency claims that the average starting salary for clerical employees in the state is more than $30,000 per year. To test this claim, she has collected a simple random sample of 100 starting salaries of clerks from across the state and found that the sample mean in $30,250.
(a) Assuming the population standard deviation is known to be $2,500 and the significance level for the test is 0.05, conduct the test. What conclusion would be reached?
(b) Referring to your answer in part (a), which of the two types of error might have been made in this case? Explain.

14. ***(7 marks)*** A retail electronics shop is thinking of mounting a sales push to get customers to buy the extended warranty available for their purchases. The sales manager believes that fewer than 10% of customers currently opt for the extra coverage. A random sample of 200 customers revealed that 12 opted for the extended warranty. Do the sample results confirm the sales manager's belief? Use $\alpha = 0.05$.

15. ***(7 marks)*** Telemarketers generally read from a prepared script when they make their sales calls. A firm decides to change this prepared script, making it both friendlier and shorter. Daily sales are recorded for a random sample of telemarketers, both before and after the script change. The average difference — using a [(before the change)-(after the change)] order of subtraction — is +4.2, with a sample size of 56. The differences have a standard deviation

of 23.4. Do the data suggest that there is a difference in daily sales before and after the script change? Use $\alpha = 0.05$.

16. ***(7 marks)*** The makers of ink cartridges for color ink-jet printers have developed a new system for storing the ink. They think the new system will result in a longer lasting product. In order to determine whether this is the case, a test was developed in which a sample of 35 of the new cartridges was selected. They were put in a printer, and test pages were run until the cartridge was empty. The same thing was done for a sample of 32 – original cartridges. The following data were observed:

New cartridge	Existing cartridge
$\bar{x}_1 = 288$ pages	$\bar{x}_2 = 279$ pages
$s_1 = 16.3$ pages	$s_2 = 15.91$ pages

Based on the sample data and a significance level equal to 0.10, determine if the new system will result in a longer lasting product.

Answers.

1. The probability that the second card drawn is a heart is 1/4.
2. There are 3003 different committees that can be formed.
3. (a) $P(A \text{ and } B) = 0.22$. (b) $P(A|B) = 0.5$. (c) $P(A \text{ or } B) = 0.94$. (d) $P(A^c \text{ and } B^c) = 0.06$.
4. (a) Yes: all probabilities are between 0 and 1, and their sum is 1. (b) He can expect to sell 1.5 dishwashers during a day. (c) The standard deviation is 1.0149 dishwashers.
5. (a) The probability that exactly 5 of the 12 children live in homes with two married parents is 3.96%. (b) The probability that at most 2 of the 12 children live in homes with two married parents is 0.0374%. (c) The expected value is 8.16 children and the standard deviation is 1.616 children.
6. (a) $P(-1.1 < z < -0.72) = 0.1001$.
7. (a) $P(29 < X < 36) = 0.1557$. (b) $P(X < 40) = 0.5$. (c) $P(X > 32) = 0.9772$.
8. (a) $\bar{X}$ is normally distributed with $\mu_{\bar{X}} = \mu = 40,000$ and $\sigma_{\bar{X}} = \sigma/\sqrt{n} = 1500/\sqrt{20}$. (b) The probability of getting an $\bar{x}$-value as low as \$38,000 is null. (c) It seems clear that the average salary of graduates from the business program is less than \$40,000 in the first year after graduation.
9. Two possible answers depending on the method used. $P(160 \leq r \leq 180) = 0.7013$ (more accurate answer) or $P(0.4 \leq \hat{p} \leq 0.45) = 0.6779$.
10. The minimal sample size is 41.
11. We are 95% confident that the average monthly rent paid by students at this College is between \$527.84 and \$558.58.
12. We are 90% confident that the proportion of all scientists who belong to professional associations is between 30.7% and 49.3%.
13. (a) There is not enough evidence to support the director's claim. (b) Type 2 error (failing to reject H_0 while it is false). Here, we may have concluded that the average starting salary for clerical employees in the state is not more than \$30,250 while it actually is.
14. The sample results confirm the sales manager's belief at a 5% significance level.
15. At a 5% significance level, we conclude that there is no difference in daily sales before and after the script change.
16. There is enough evidence at a 10% significance level, to conclude that the new system will result is a longer lasting product.

Formulas

Counting $P_{n,r} = \dfrac{n!}{(n-r)!}$ $C_{n,r} = \dfrac{n!}{r!(n-r)!}$

Random variable $\mu = E(X) = \sum_x xP(x)$ $\sigma = \sqrt{\sum_x (x-\mu)^2 P(x)} = \sqrt{[\sum_x x^2 P(x)] - \mu^2}$

Binomial variable $P(x) = \binom{n}{x} p^x q^{n-x} = \dfrac{n!}{x!(n-x)!} p^x q^{n-x}$ $\mu = np$ $\sigma = \sqrt{npq}$

Sample mean and standard deviation $\bar{x} = \dfrac{\sum x}{n}$ $s = \sqrt{\dfrac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}}$

Z-scores: For parent population: $z = \dfrac{x-\mu}{\sigma}$ $x = \sigma z + \mu$

For sampling distribution: $z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

Statistics for one sample mean: $E = z_c \dfrac{\sigma}{\sqrt{n}}$ $z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}}$ $n \geq \left(\dfrac{z_c \sigma}{E}\right)^2$

$E = t_c \dfrac{s}{\sqrt{n}}$ $t = \dfrac{\bar{x}-\mu}{s/\sqrt{n}}$ $n \geq \left(\dfrac{t_c s}{E}\right)^2$

Statistics for one sample proportion: $\hat{p} = \dfrac{r}{n}$ $\hat{q} = 1 - \hat{p}$

$E = z_c \sqrt{\dfrac{\hat{p}\hat{q}}{n}}$ $z = \dfrac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$

Statistics for two sample means, dependent samples (or paired differences):

$\bar{d} = \dfrac{\sum d}{n}$, where $d = x_1 - x_2$ $s_d = \sqrt{\dfrac{\sum d^2 - \frac{(\sum d)^2}{n}}{n-1}}$

$E = t \dfrac{s_d}{\sqrt{n}}$ $t = \dfrac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$

Statistics for two sample means, independent samples:

$E = z\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$ $z = \dfrac{(\bar{x_1} - \bar{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$

$E = t\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$ $t = \dfrac{(\bar{x_1} - \bar{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

Standard Normal Probabilities

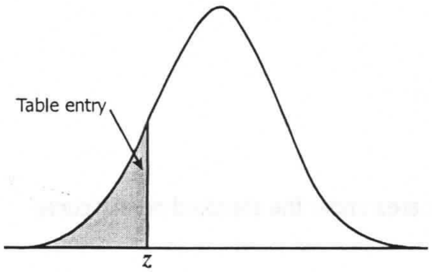


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

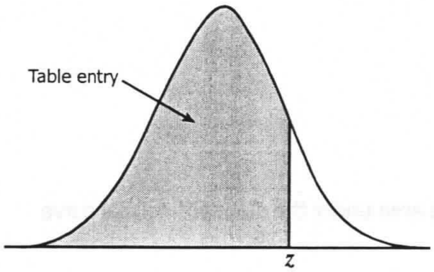
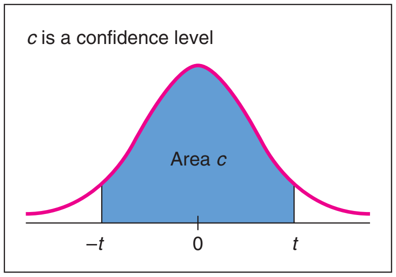
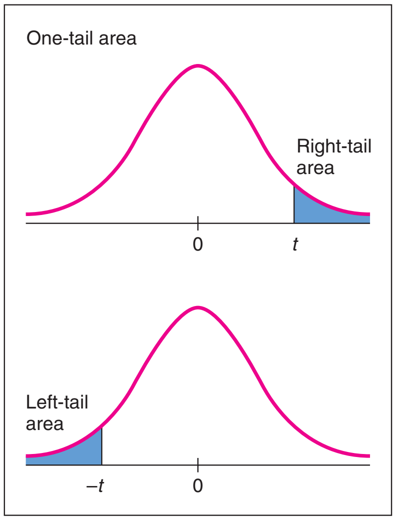


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998





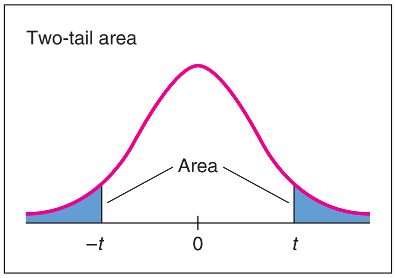


TABLE 6 Critical Values for Student's *t* Distribution

one-tail area	0.250	0.125	0.100	0.075	0.050	0.025	0.010	0.005	0.0005
two-tail area	0.500	0.250	0.200	0.150	0.100	0.050	0.020	0.010	0.0010
d.f. \ *c*	0.500	0.750	0.800	0.850	0.900	0.950	0.980	0.990	0.999
1	1.000	2.414	3.078	4.165	6.314	12.706	31.821	63.657	636.619
2	0.816	1.604	1.886	2.282	2.920	4.303	6.965	9.925	31.599
3	0.765	1.423	1.638	1.924	2.353	3.182	4.541	5.841	12.924
4	0.741	1.344	1.533	1.778	2.132	2.776	3.747	4.604	8.610
5	0.727	1.301	1.476	1.699	2.015	2.571	3.365	4.032	6.869
6	0.718	1.273	1.440	1.650	1.943	2.447	3.143	3.707	5.959
7	0.711	1.254	1.415	1.617	1.895	2.365	2.998	3.499	5.408
8	0.706	1.240	1.397	1.592	1.860	2.306	2.896	3.355	5.041
9	0.703	1.230	1.383	1.574	1.833	2.262	2.821	3.250	4.781
10	0.700	1.221	1.372	1.559	1.812	2.228	2.764	3.169	4.587
11	0.697	1.214	1.363	1.548	1.796	2.201	2.718	3.106	4.437
12	0.695	1.209	1.356	1.538	1.782	2.179	2.681	3.055	4.318
13	0.694	1.204	1.350	1.530	1.771	2.160	2.650	3.012	4.221
14	0.692	1.200	1.345	1.523	1.761	2.145	2.624	2.977	4.140
15	0.691	1.197	1.341	1.517	1.753	2.131	2.602	2.947	4.073
16	0.690	1.194	1.337	1.512	1.746	2.120	2.583	2.921	4.015
17	0.689	1.191	1.333	1.508	1.740	2.110	2.567	2.898	3.965
18	0.688	1.189	1.330	1.504	1.734	2.101	2.552	2.878	3.922
19	0.688	1.187	1.328	1.500	1.729	2.093	2.539	2.861	3.883
20	0.687	1.185	1.325	1.497	1.725	2.086	2.528	2.845	3.850
21	0.686	1.183	1.323	1.494	1.721	2.080	2.518	2.831	3.819
22	0.686	1.182	1.321	1.492	1.717	2.074	2.508	2.819	3.792
23	0.685	1.180	1.319	1.489	1.714	2.069	2.500	2.807	3.768
24	0.685	1.179	1.318	1.487	1.711	2.064	2.492	2.797	3.745
25	0.684	1.198	1.316	1.485	1.708	2.060	2.485	2.787	3.725
26	0.684	1.177	1.315	1.483	1.706	2.056	2.479	2.779	3.707
27	0.684	1.176	1.314	1.482	1.703	2.052	2.473	2.771	3.690
28	0.683	1.175	1.313	1.480	1.701	2.048	2.467	2.763	3.674
29	0.683	1.174	1.311	1.479	1.699	2.045	2.462	2.756	3.659
30	0.683	1.173	1.310	1.477	1.697	2.042	2.457	2.750	3.646
35	0.682	1.170	1.306	1.472	1.690	2.030	2.438	2.724	3.591
40	0.681	1.167	1.303	1.468	1.684	2.021	2.423	2.704	3.551
45	0.680	1.165	1.301	1.465	1.679	2.014	2.412	2.690	3.520
50	0.679	1.164	1.299	1.462	1.676	2.009	2.403	2.678	3.496
60	0.679	1.162	1.296	1.458	1.671	2.000	2.390	2.660	3.460
70	0.678	1.160	1.294	1.456	1.667	1.994	2.381	2.648	3.435
80	0.678	1.159	1.292	1.453	1.664	1.990	2.374	2.639	3.416
100	0.677	1.157	1.290	1.451	1.660	1.984	2.364	2.626	3.390
500	0.675	1.152	1.283	1.442	1.648	1.965	2.334	2.586	3.310
1000	0.675	1.151	1.282	1.441	1.646	1.962	2.330	2.581	3.300
∞	0.674	1.150	1.282	1.440	1.645	1.960	2.326	2.576	3.291

For degrees of freedom *d.f.* not in the table, use the closest *d.f.* that is *smaller*.