Dawson College
Mathematics Department
Final Examination
201-015-50 (Remedial Activities For Sec. V Math)
Thursday, May 19, 2016
Time: 9:30 – 12:30 (3 hours)

Student Name: ______________________________

Student I.D. #: ______________________________

Teacher: ______________________________

Instructors: G. Chu and R. Acteson

Instructions:

- Print your name and student I.D. number in the space provided above.
- Attempt all questions.
- All questions are to be answered directly on the examination paper.
- Translation and regular dictionaries are permitted.
- Students are only permitted to use the Sharp EL-531XG or -531X calculator.
- This examination consists of 19 questions on pages 2 to 10.
- Please ensure that you have a complete exam package before starting.

1. [Marks 5] Divide by long division. $\dfrac{6x^4 - 7x^2 + 1}{2x^2 - 1}$

$$\begin{array}{r l}
 & 3x^2 \quad -2 \quad -\frac{1}{2x^2-1} \\
2x^2-1 \,\big) & 6x^4 \quad -7x^2 \quad +1 \\
 & -(6x^4 \quad -3x^2) \\
 & 0 \quad -4x^2 \quad +1 \\
 & -(-4x^2 \quad +2) \\
 & -1
\end{array}$$

$$\boxed{3x^2 - 2 - \frac{1}{2x^2-1}}$$

2. [Marks 5] Divide and simplify the expression. $\dfrac{2x^3 + 8x^2 - 42x}{x^2 - 9} \div \dfrac{4x^2 + 28x}{x^2 + 8x + 15}$

$$= \frac{\cancel{2x}(x^2+4x-21)}{(x-3)\cancel{(x+3)}} \cdot \frac{(x+5)\cancel{(x+3)}}{\cancel{4x}_{2}(x+7)}$$

$$= \frac{\cancel{(x+7)}\cancel{(x-3)}(x+5)}{2\cancel{(x-3)}\cancel{(x+7)}} = \boxed{\frac{x+5}{2}}$$

3. [Marks 5] Rationalize the denominator and simplify. $\dfrac{(3+\sqrt{6})(\sqrt{6}-2)}{6-\sqrt{6}}$

$$= \left(\frac{3\sqrt{6} - 6 + 6 - 2\sqrt{6}}{6-\sqrt{6}}\right)\left(\frac{6+\sqrt{6}}{6+\sqrt{6}}\right)$$

$$= \frac{\sqrt{6}(6+\sqrt{6})}{36-6} = \frac{6\sqrt{6}+6}{30} = \frac{\cancel{6}(\sqrt{6}+1)}{\cancel{30}\,5} = \boxed{\frac{\sqrt{6}+1}{5}}$$

4. [Marks 5] Solve the equation. $\frac{3x-1}{4x+7} = 1 - \frac{6}{x+7}$ $LCD = (4x+7)(x+7)$

$$\left(\frac{3x-1}{4x+7}\right)\left(\frac{x+7}{x+7}\right) + \left(\frac{6}{x+7}\right)\left(\frac{4x+7}{4x+7}\right) = 1\left(\frac{4x+7}{4x+7}\right)\left(\frac{x+7}{x+7}\right)$$

$$\cancel{(LCD)}\ \frac{3x^2+21x-x-7+24x+42}{\cancel{(4x+7)(x+7)}} = \frac{4x^2+28x+7x+49}{\cancel{(4x+7)(x+7)}}\ \cancel{(LCD)}$$

$$3x^2+44x+35 = 4x^2+35x+49$$

$$0 = x^2-9x+14$$

$$0 = (x-2)(x-7)$$

$$\begin{matrix} \shortparallel & \shortparallel \\ 0 & 0 \end{matrix}$$

$$\boxed{x=2 \text{ or } x=7}$$

5. [Marks 5] Solve the equation. $\sqrt{5x+1} - 1 = x$

$$\left(\sqrt{5x+1}\right)^2 = (x+1)^2$$

$$5x+1 = x^2+2x+1$$

$$0 = x^2-3x$$

$$0 = x(x-3)$$

$$\begin{matrix} \shortparallel & \shortparallel \\ 0 & 0 \end{matrix}$$

$$\boxed{x=0 \text{ or } x=3}$$

6. [Marks 5] Find an equation of a circle with $(-6,2)$ and $(4,4)$ as endpoints of the diameter.

Center of circle = midpoint of endpoints $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$\boxed{= \left(\frac{-6+4}{2}, \frac{2+4}{2}\right) = (-1,3)}$$

radius of circle = distance from center to point on circumference

$$r = \sqrt{(4+1)^2+(4-3)^2} = \sqrt{5^2+1^2} = \sqrt{26}$$

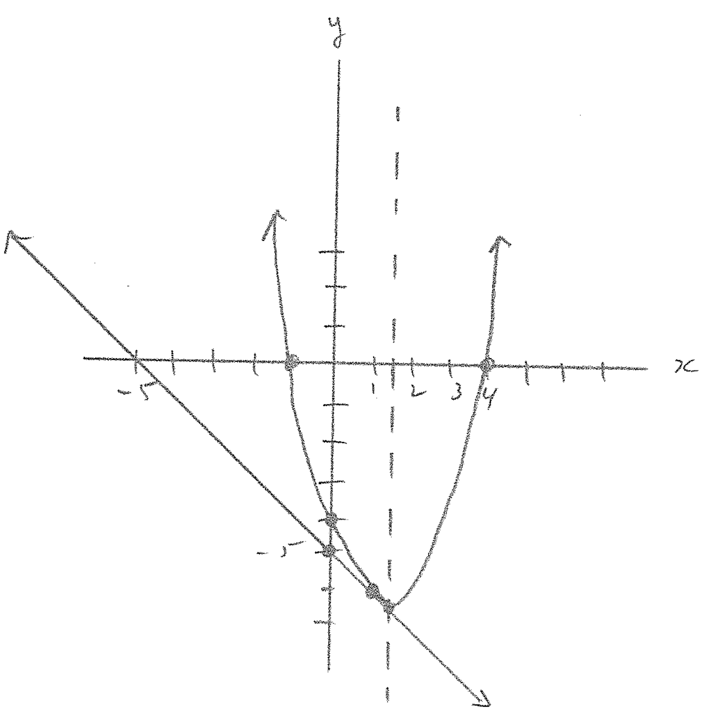
Equation of circle $(x-h)^2+(y-k)^2=r^2$

$$\boxed{(x+1)^2+(y-3)^2=26}$$

7. [Marks 5] Graph $y = -x - 5$ and $y = x^2 - 3x - 4$ and find their intersections.

axis of symmetry $x = -\frac{b}{2a} = \frac{3}{2}$

$$y = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) - 4 = \frac{9}{4} - \frac{9}{2} - 4$$
$$= \frac{9}{4} - \frac{18}{4} - \frac{16}{4} = \frac{-24}{4} = -6.25$$



$y = -x-5$
$y = x^2-3x-4$

$$-x-5 = x^2-3x-4$$
$$0 = x^2-2x+1$$
$$0 = (x-1)(x-1)$$

$x=1$

$$y = -1-5 = 6$$

point of interception

$(x,y) = \underline{(1,6)}$

8. Consider the functions $f(x) = \sqrt[3]{x+2} + 1$ and $g(x) = \dfrac{3}{x^2 - x}$.

a. [Marks 3] Find $(g \circ f)(-10)$

b. [Marks 2] State the domain of $g(x)$

c. [Marks 5] Find the inverse, $f^{-1}(x)$

a) $(g \circ f)(-10) = g(f(-10)) = g(\sqrt[3]{-10+2} + 1) = g(\sqrt[3]{-8} + 1) = g(-2+1)$

$= g(-1) = \dfrac{3}{(-1)^2 - (-1)} = \dfrac{3}{1+1} = \boxed{3/2}$

b) domain of $g(x)$ $\rightarrow$ $g(x) = \dfrac{3}{x(x-1)}$, cannot divide by zero.

Domain: $\mathbb{R} \setminus \{0, 1\}$ $\quad x(x-1) \neq 0$ $\quad x \neq 0$ and $x \neq 1$

$(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

c) $y = \sqrt[3]{x+2} + 1$

$x = \sqrt[3]{y+2} + 1$

$x - 1 = \sqrt[3]{y+2}$

$(x-1)^3 = y + 2$

$y = (x-1)^3 - 2$

$f^{-1}(x) = (x-1)^3 - 2$

9. [Marks 5] Express as a single logarithm with coefficient one: $2\log x - 3\log y + 4\log v - \frac{1}{2}\log w$

$= \log x^2 - \log y^3 + \log v^4 - \log\sqrt{w}$

$= \log\left(\frac{x^2 v^4}{y^3\sqrt{w}}\right)$

10. [Marks 5] Solve for x: $\ln(4x-10) - \ln(x-2) = \ln(x-1)$

$\ln(4x-10) = \ln(x-1) + \ln(x-2)$

$\ln(4x-10) = \ln(x-1)(x-2)$

$4x-10 = x^2-3x+2$

$0 = x^2-7x+12$

$0 = (x-3)(x-4)$

$\boxed{x=3 \text{ or } x=4}$

11. [Marks 5] Solve for x: $8^{(2x+1)} = 6^{(x-3)}$

$\ln 8^{2x+1} = \ln 6^{x-3}$

$(2x+1)\ln 8 = (x-3)\ln 6$

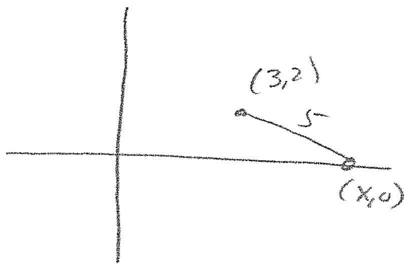
$(2x+1)(2.07944) = (x-3)(1.79176)$

$4.1589x + 2.07944 = 1.79176x - 5.3752$

$2.36712x = -7.45472$

$\boxed{x = -3.149}$

12. [Marks 5] Find a point on the x-axis that is 5 units from the point $(3,2)$.



$$\text{distance} = \sqrt{(x-3)^2+(0-2)^2} = 5$$

$$\left(\sqrt{(x-3)^2+4}\right)^2 = (5)^2$$

$$(x-3)^2+4 = 25$$

$$(x-3)^2 = 21$$

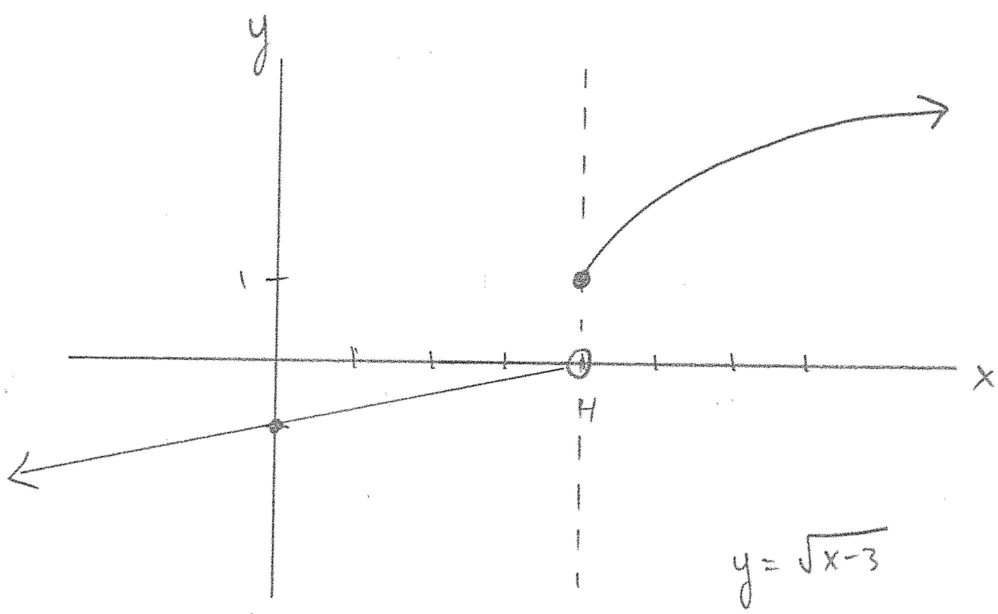
$$x-3 = \pm\sqrt{21}$$

$$\boxed{x = 3 \pm \sqrt{21}}$$

13. [Marks 5] Graph the function and state the domain and range of the function.

$$f(x) = \begin{cases} \dfrac{x-4}{4} & \text{if } x < 4 \\ \sqrt{x-3} & \text{if } x \geq 4 \end{cases}$$

Domain: $\mathbb{R}$
Range: $(-\infty, 0) \cup [1, \infty)$



$$y = \frac{x-4}{4} = \frac{1}{4}x - 1$$

x	y
4	0
0	-1

14. [Marks 5] If $f(x) = x^2 - 2x$, then find $\dfrac{f(x+h) - f(x)}{h}$ and simplify.

$$f(x+h) = (x+h)^2 - 2(x+h) = x^2 + 2xh + h^2 - 2x - 2h$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\cancel{x^2} + 2xh + h^2 - \cancel{2x} - 2h - (\cancel{x^2} - \cancel{2x})}{h}$$

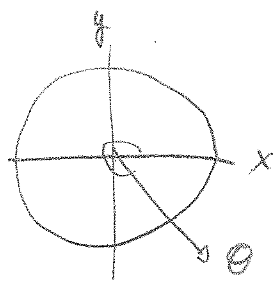
$$= \frac{2xh + h^2 - 2h}{h}$$

$$= \frac{\cancel{h}(2x + h - 2)}{\cancel{h}} = \boxed{2x + h - 2}$$

15. [Marks 5] Find the value of $\sec\theta + \cos(2\theta)$ if $\cos\theta > 0$ and $\tan\theta = -\dfrac{3}{2}$.

θ in II or IV

$\hookrightarrow \theta$ in I or IV



θ in IV quadrant

$$\tan\theta = -\frac{3}{2} = \frac{y}{x} \rightarrow \left.\begin{array}{l} y = -3 \\ x = 2 \end{array}\right\} r = ?$$

$$x^2 + y^2 = r^2$$
$$4 + 9 = r^2 \rightarrow r = \sqrt{13}$$

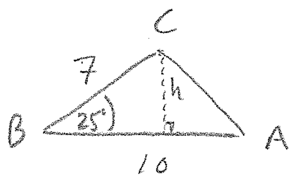
$$\sec\theta + \cos(2\theta) = \frac{1}{\cos\theta} + \cos^2(\theta) - \sin^2(\theta)$$

$$= \frac{r}{x} + \left(\frac{x}{r}\right)^2 - \left(\frac{y}{r}\right)^2$$

$$= \frac{\sqrt{13}}{2} + \left(\frac{2}{\sqrt{13}}\right)^2 - \left(\frac{-3}{\sqrt{13}}\right)^2$$

$$= \frac{\sqrt{13}}{2} + \frac{4}{13} - \frac{9}{13} = \boxed{\frac{\sqrt{13}}{2} - \frac{5}{13}}$$

16. [Marks 5] Find the area of the triangle ABC, given that $a = 7$ cm, $B = 25°$ and $c = 10$ cm.



$\sin 25° = \frac{h}{7} \rightarrow h = 7 \sin 25° \approx 2.958$ cm

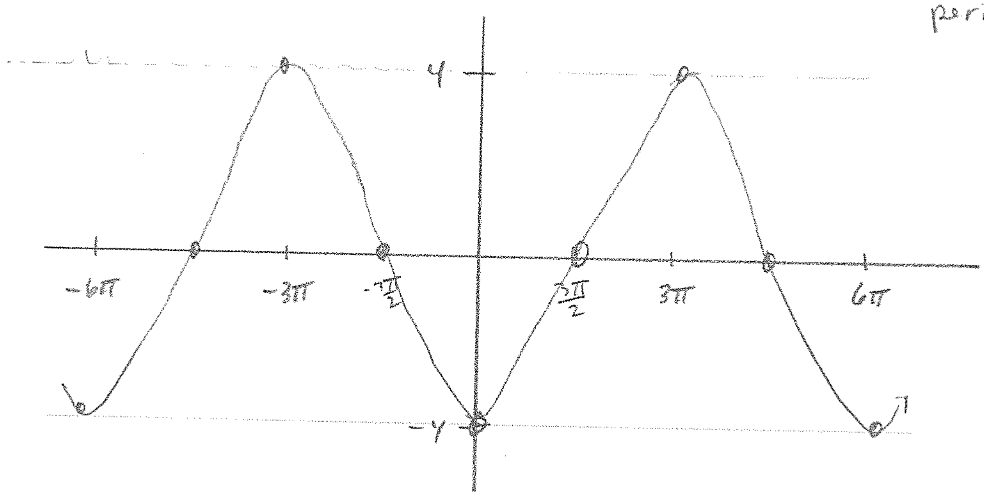
$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(10)(2.958)$

$\approx \underline{14.79 \text{ cm}^2}$

17. [Marks 5] Graph $y = -4\cos\left(\frac{x}{3}\right)$ over two periods. State the amplitude and period of the function.

$\text{Amplitude} = |-4| = 4$

$\text{period} = \frac{2\pi}{b} = \frac{2\pi}{1/3} = 6\pi$



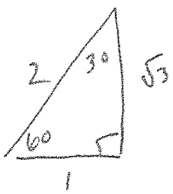
18. [Marks 5] Solve for x, giving the exact solution where possible, $0 \le x < 2\pi$.

$2\sin\theta + \sqrt{3} = 0$

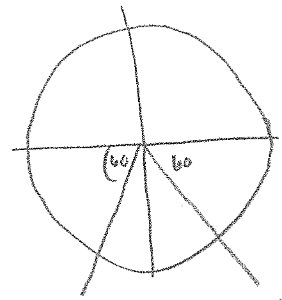
$2\sin\theta = -\sqrt{3}$

$\sin\theta = -\frac{\sqrt{3}}{2}$

$60^\circ = \frac{\pi}{3}$ rad



$\sin 60^\circ = \frac{\sqrt{3}}{2}$



$\theta = 240^\circ$ $\quad$ $\theta = 300^\circ$

$\boxed{\theta = \frac{4\pi}{3} \text{ rad} \text{ and } \theta = \frac{5\pi}{3} \text{ rad}}$

19. [Marks 5] For which value(s) of x are the vectors $\vec{u} = (x, 4)$ and $\vec{w} = (-2, 3)$ perpendicular.

$u \perp v \iff u \circ v = 0$

$u \circ v = (x, 4) \circ (-2, 3)$

$= -2x + 4(3)$

$= -2x + 12$

$-2x + 12 = 0$

$12 = 2x$

$x = 6$

Information Sheet

$$a^3+b^3=(a+b)\left(a^2-ab+b^2\right)$$
$$a^3-b^3=(a-b)\left(a^2+ab+b^2\right)$$

$$\log_a(xy)=\log_a(x)+\log_a(y)$$
$$\log_a\left(\frac{x}{y}\right)=\log_a(x)-\log_a(y)$$
$$\log_a(x^p)=p\log_a(x)$$
$$\log_a(1)=0$$
$$\log_a(a)=1$$
$$\log_a(x)=\frac{\log_b(x)}{\log_b(a)}$$

$$\sin^2\theta+\cos^2\theta=1$$
$$1+\tan^2\theta=\sec^2\theta$$
$$1+\cot^2\theta=\csc^2\theta$$

$$\sin(A+B)=\sin A\cos B+\cos A\sin B$$
$$\sin(A-B)=\sin A\cos B-\cos A\sin B$$
$$\cos(A+B)=\cos A\cos B-\cos A\cos B$$
$$\cos(A-B)=\cos A\cos B+\cos A\cos B$$

$$\sin(2A)=2\sin A\cos A$$
$$\cos(2A)=\cos^2A-\sin^2A$$
$$\cos(2A)=2\cos^2A-1$$
$$\cos(2A)=1-2\sin^2A$$

If $y=\arcsin x$ then $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$

If $y=\arccos x$ then $0\le y\le\pi$

If $y=\arctan x$ then $-\frac{\pi}{2}<y<\frac{\pi}{2}$