**DAWSON COLLEGE**
**Mathematics Department**
**Final Examination**
**Linear Algebra**
**201–NYC–05 (Commerce)**
**Fall 2019**

1. a) (5 marks) Use Gauss-Jordan elimination to find the general solution of the system.
   b) (1 mark) Find a particular solution in which $x_1 = 3$ and $x_3 = -5$.

$$x_1 + x_2 - 2x_3 + x_4 = 7$$
$$2x_1 + x_2 - 2x_3 = 8$$
$$3x_1 + 2x_2 - 4x_3 + x_4 = 15$$

2. (6 marks) Given the system of linear equations $\begin{cases} 2x + y + z = 1 \\ -x + 2y - 2z = -1 \\ 3x - 2y + 5z = 5 \end{cases}$.

a) Use the adjoint matrix to find the inverse of the coefficient matrix.
b) Use the inverse of the coefficient matrix to solve the system.

3. (3 marks) Use Cramer's rule to solve the system from question #2 **<u>for "z" only</u>**.

4. (3+1+4 marks) Let $A = \begin{bmatrix} 7 & -3 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ 5 & -2 \\ 1 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$.

a. Calculate $tr(A^{-2} - 3B^T B + 4C^{2019})$

b. Does the system $B^T X = 0$ have a nontrivial solution? Justify your answer without solving the system.

c. Solve for $X$: $\left(AX^T + 2I^{-1}\right)^{-1} = A^2$

5. (3 marks) Determine the values of $a$ such that the system has
   1) a unique solution, 2) infinitely many solutions, 3) no solution :

$$\begin{cases} x + 2y + 4az = 3 \\ y - 7az = 2 \\ -x - 3y + \left(a^2 + 2a\right)z = a - 6 \end{cases}$$

6. (3 marks) Let $A = \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}$. Write $A^{-1}$ as a product of elementary matrices.

7. (3 marks) Simplify $B^2\left(3A^{-1}B\right)^{-1} \cdot \left(D^T A\right)^{-1} \cdot \left(3C^T D\right)^T - CD^0 B$.

8. (4 marks) Evaluate the determinant by row reduction

$$\begin{vmatrix} 3 & -2 & -4 & 2 \\ -2 & 3 & 1 & -3 \\ 1 & -1 & 7 & 2 \\ 3 & -4 & -2 & 6 \end{vmatrix}$$

9. (4+4+4 marks) If $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $B$ is a $3 \times 3$ with $\det(A) = -2$ and $\det(B) = 4$, then find

a) $\begin{vmatrix} d+mg & 3a+g & -g+a \\ e+mh & 3b+h & -h+b \\ f+mi & 3c+i & -i+c \end{vmatrix}$, where $m$ is a real number.

b) $\det\left(-\left(2B^3\right)^{-1}\cdot\left(-B^T A^{-1}\right)^4\right)$

c) $\det\left(\left(A^{-1}B\right)^{-1}+B^{-1}\cdot adj(A^{-1})\right)$

10. (3+3+3+3 marks) Let $\vec{u}=(3,\ 2,-1)$, $\vec{v}=(1,\ 2,\ 1)$ and $\vec{w}=-\vec{i}+\vec{j}-2\vec{k}$

a) Find the area of the triangle determined by $\vec{u}+\vec{w}$ and $\vec{u}-\vec{v}$.

b) Find a vector of length 5 perpendicular to both vectors $\vec{u}+\vec{w}$ and $\vec{u}-\vec{v}$.

c) Find $Proj_{\overline{u-v}}\left(\vec{u}+\vec{w}\right)$

d) Find the value(s) of $k$ such that the vector $\vec{v}+k\vec{w}$ is perpendicular to $\vec{v}+k\vec{u}$.

11. (3 marks) Let $\left[\left(\vec{a}-\vec{c}\right)\times\left(\vec{a}+\vec{c}\right)\right]\cdot\left(\vec{a}-\vec{b}\right)=-4$. Find the volume of the parallelepiped with edges $\vec{a}, \vec{b}, \vec{c}$.

12. (1+3 marks) a) Determine whether the planes $x+2y-z=2$ and $2x+3y-5z=1$ are parallel.
b) If they are parallel, then find the distance between them. If the planes are not parallel, then find the parametric equations of the line of intersection of the planes.

13. (12 marks) Given a point $A(2,-1,\ 5)$ and two lines L1: $\begin{cases} x=7-6t \\ y=2-3t \\ z=6+4t \end{cases}$ and L2: $\begin{cases} x=-2+u \\ y=5-2u \\ z=1+3u \end{cases}$.

a. Find the distance from a point $A(2,-1,\ 5)$ and line L1.
b. Find the equation of the plane which contains point $A$ and line L1.
c. Find a point on the line L2 which is closest to the point $A$.
d. Find the point of intersection of the lines L1 and L2.

14. (4 marks) Show that L1: $\begin{cases} x=2+t \\ y=1+4t \\ z=-3-2t \end{cases}$ and L2: $\begin{cases} x=-1+u \\ y=\ 3-2u \\ z=\ 1\ +\ u \end{cases}$ are skew lines and find the distance between them.

15. (3 marks) Determine whether the following statement is true or false. <u>If the statement is true, then prove it.</u> <u>If the statement is false, then provide a counter-example that shows that the statement is not true.</u>

"If $A$ and $B$ are skew-symmetric $2\times 2$ matrices then matrix $A^2+2AB+B^2$ will be symmetric".

16. (7 marks) Maximize $P=2x_1-3x_2+6x_3$ subject to $\begin{cases} 4x_1+2x_2-x_3\le 3 \\ x_1-x_2+\ x_3\le 2 \\ 5x_1-2x_2+2x_3\le 6 \end{cases}$ $\left(x_1,x_2,x_3\ge 0\right)$

17. (7 marks) Minimize $C=2x_1+13x_2$ subject to $\begin{cases} 2x_1+x_2\ge 5 \\ x_1+3x_2\ge 7 \\ x_1+2x_2\ge 4 \end{cases}$ $\left(x_1,x_2\ge 0\right)$

## Answers

**1**. a) $x_1 = 1+t,\ x_2 = 6+2s-2t,\ x_3 = s,\ x_4 = t.$ b) $x_1 = 3,\ x_2 = -8,\ x_3 = -5,\ x_4 = 2.$

**2**. a) $A^{-1} = \begin{bmatrix} \frac{6}{7} & -1 & -\frac{4}{7} \\ -\frac{1}{7} & 1 & \frac{3}{7} \\ -\frac{4}{7} & 1 & \frac{5}{7} \end{bmatrix}$; b) $x=-1,\ y=1,\ z=2.$

**3**. $z = 2$

**4.** a) $-94$; b) Yes, it does. c) $X = \begin{bmatrix} 29 & 70 \\ -105 & -251 \end{bmatrix}.$

**5.** 1) $a \neq 0,\ a \neq 1$; 2) $a = 1$; 3) $a = 0$

**6.** $A^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$. *Other possible answers.*

**7.** $BC - CB$

**8.** 80.

**9.** a) $-8$; b) $-\frac{1}{32}$; c) $-\frac{1}{16}$

**10.** a) $\sqrt{19}$ ; b) $\left(-\frac{15}{\sqrt{19}}, -\frac{5}{\sqrt{19}}, -\frac{15}{\sqrt{19}}\right)$; c) $\left(\frac{5}{2}, 0, -\frac{5}{2}\right)$; d) $k=-2,\ k=-3.$

**11. 2**

**12.** a) not parallel; b) $x = -4+7t,\ y = 3-3t,\ z = t$ .

**13.** a) $\frac{\sqrt{910}}{\sqrt{61}}$ ; b) $-15x+26y-3z+71=0$; c) $(0,1,7)$; d) $(1,-1,10)$.

**14.** $2\sqrt{5}$

**15.** True;

**16.** $P = 27$, $x_1 = 0,\ x_2 = 5,\ x_3 = 7.$

**17.** $C = 14$, $x_1 = 7,\ x_2 = 0.$