Dawson college
Department of Mathematics
201 NYB 05
Winter 2019
9:30 AM to 12:30 PM
5/24/19
Final exam-Version A

Student Name: ______________________
Student ID: ______________________

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Instructions

1. Write your name and ID number at the top, right of this page.
2. Please note that there are 14 questions.
3. Answer the questions in the spaces provided. If you require additional space to answer a question, please use the back of the page and refer to this page in your solutions.
4. **Your grade will be influenced by how clearly you express your ideas, and how well you organize your solutions. Numerical answers should be in exact values.**
5. **DO NOT WRITE FORMULAS ON THIS COVER PAGE.**

Marking Scheme:

Question	Out of	Score
1	6	
2	16	
3	8	
4	6	
5	6	
6	6	
7	6	
8	10	
9	3	
10	3	
11	3	
12	16	
13	5	
14	6	
$\sum$	100	

1. Use a limit of Riemann sums to compute the exact value of the definite integral (6 marks)

$$\int_{-3}^{1} (4 - x^2)\, dx$$

$$\sum_{i=1}^{n} c = cn, \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(2n+1)(n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

2. Evaluate the following integrals (16 marks)

(a)

$$\int \cos^3 x \ln(\sin x)\, dx$$

(b)

$$\int \left(\frac{e^x}{e^{3x} + e^x} \right) dx$$

(c)

$$\int \left(\frac{\cos x + \sin^3 x}{\sec x} \right) dx$$

(d)

$$\int \left(\frac{(x-2)^2}{\sqrt{5+4x-x^2}} \right) dx$$

3. (8 marks)

(a) Given $g(x) = \int_0^{2x} \sqrt{e^t - \frac{1}{4}}\, dt$, find $g'(x)$.

(b) Use your answer in (a) to find the arc length of $g(x)$ over $[1, 2]$.

4. Write the following limit of the Riemann sums as a definite integral (6 marks)

(a)

$$\lim_{n\to\infty} \frac{1}{n}\left[\left(3+\frac{1}{n}\right)^3 + \left(3+\frac{2}{n}\right)^3 + \cdots + \left(3+\frac{n}{n}\right)^3\right]$$

(b) By evaluating the definite integral from (a), conclude the value of the limit.

5. Determine whether the following improper integral is convergent or divergent, if it is convergent, find its value.

(6 marks)

$$\int_0^{+\infty} \left(\frac{1}{(1+x)\sqrt{x}}\right) dx$$

6. Determine whether the following improper integral is convergent or divergent, if it is convergent, find its value

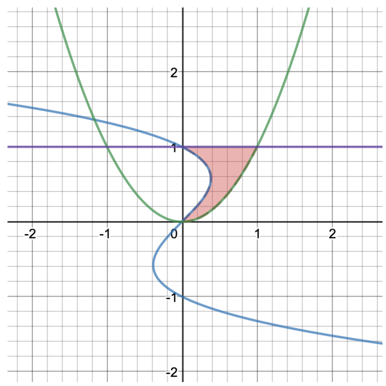
(6 marks)

$$\int_{-1}^{0} \left(\frac{e^{\frac{1}{x}}}{x^3} \right) dx$$

7. A certain spring system is damped so the position of the mass as a function of time is $y(t) = 2e^{-t}\cos t$. Find the average value of y over the time interval $[0,2\pi]$. (6 marks)

8. Let R be the shaded region bounded by $x = y - y^3$, $y = x^2$, $y = 1$ and $x \geq 0$. See below.

(10 marks)



(a) Set up an integral or integrals that express the area of R (DO NOT EVALUATE).

(b) Use the most convenient method(Disk method or Shell method) to set up the integral to find the volume obtained by rotating region R around $x = 2$ (DO NOT EVALUATE)

(c) Use the most convenient method(Disk method or Shell method) to set up the integral to find the volume obtained by rotating region R around x axis (DO NOT EVALUATE)

9. Find the limit of the sequence $\{\left(\frac{1}{n+1}\right)\arctan n\}$ (3 marks)

10. Find the n^{th} term of the sequence (3 marks)

$$\sqrt{3},\ \sqrt{6},\ 2\sqrt{3},\ 2\sqrt{6},\ 4\sqrt{3},\ 4\sqrt{6},\ldots$$

11. Determine whether the series is convergent or divergent, if it is convergent find its sum

(3 marks)

$$\sum_{n=1}^{+\infty} \left(\frac{1}{2^{n+1}} - \frac{1}{2^n} \right)$$

12. Determine whether the following series is absolutely convergent , conditionally convergent or divergent, state clearly the test you have used in each problem (16 marks)

(a)

$$\sum_{n=2}^{+\infty} \left(\frac{(-1)^n}{n(\ln n)^2} \right)$$

(b)

$$\sum_{n=1}^{+\infty} \left(\frac{n}{n+2} \right)^n$$

(c)

$$\sum_{n=1}^{+\infty} \left(2^{n-1} 3^{1-n} \cos n \right)$$

(d)

$$\sum_{n=1}^{+\infty} \left(\frac{5^n}{(2n)!} \right)$$

13. Find the radius of convergence and the interval of convergence of (5 marks)

$$\sum_{n=0}^{+\infty} \left(\frac{(-1)^n}{(2n+1)} (2x-1)^{2n+1} \right)$$

14. Find the Taylor series of $f(x) = \ln(x - 1)$ about $a = 2$. (6 marks)

Answers

1. $20/3$

2. (a) $$\left(\sin x - \frac{\sin^3 x}{3}\right)\ln(\sin x) - \sin x + \frac{\sin^3 x}{9} + C$$

(b) $$x - \frac{1}{2}\ln(1+e^{2x}) + C$$

(c) $$\frac{1}{2}x + \frac{1}{4}\sin(2x) + \frac{\sin^4(x)}{4} + C$$

(d) $$\frac{9}{2}\arcsin\left(\frac{x-2}{3}\right) - \frac{(x-2)\sqrt{5+4x-x^2}}{2} + C$$

3. (a) $$g'(x) = 2\sqrt{e^{2x} - \frac{1}{4}}$$

(b) $$L = 2[e^2 - e]$$

4. a. $\int_0^1 (3+x)^3 dx$ or $\int_3^4 x^3 dx$

b. $175/4$

5. π Conv.

6. $-2/e$ Conv.

7. $f_{ave} = \frac{1-e^{-2\pi}}{2\pi}$

8. a. $A = \int_{y=0}^{1} [\sqrt{y} - (y-y^3)]\, dy$

b. $V = \pi \int_{y=0}^{1} [(2-(y-y^3))^2 - (2-\sqrt{y})^2]\, dy$

c. $V = 2\pi \int_{y=0}^{1} y(\sqrt{y} - (y-y^3))\, dy$

9. 0

10. $a_n = \sqrt{3}\left(\sqrt{2}\right)^{n-1}$

11. $-1/2$

12. a. abs. conv.

b. div

c. abs. conv.

d. abs conv

13. $R = \frac{1}{2}$, interval of conv.
$[0, 1]$

14. $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}(x-2)^n$, $|x-2|<1$