

**DAWSON COLLEGE
MATHEMATICS DEPARTMENT**

FINAL EXAMINATION

201-NYA-05 CALCULUS I – SCIENCE

THURSDAY DECEMBER 20th 2018

(1) **(4+4+4 marks)** Find the limit, if it exists. If the limit does not exist explain why.

Do not use L'Hôpital's Rule.

(a) $\lim_{x \rightarrow 1} \frac{\sqrt{x+3} - 2}{2 - 2x^2}$

(b) $\lim_{x \rightarrow 1^-} (x-1) \sin\left(\frac{3}{x-1}\right)$

(c) $\lim_{x \rightarrow -\infty} \left(\frac{x-10}{\sqrt{x^2+11x}} \right)$

(2) **(5 marks)** Find c if the following limit exists and in this case evaluate the limit:

$$\lim_{x \rightarrow 3} \frac{\frac{3}{x+c} - 1}{x-3}$$

(3) **(5 marks)** Use only the limit definition of the derivative to calculate $f'(x)$ if $f(x) = x(2x+1)$.

(4) **(5 marks)** Given:

$$f(x) = \begin{cases} \frac{x^2 - 2x + 1}{x^2 + 4x - 5} & x < 1 \\ 0 & x = 1 \\ \frac{\sin(x-1)}{2x-2} & x > 1 \end{cases}$$

Using the definition of continuity of a function find the point(s) of discontinuity of the function and specify their types.

(5) **(3 marks)** Find the values of c where the functions $y = f(x) = 2x^3$ and $y = g(x) = 2x - 2x^2$ have parallel tangent lines at $x = c$.

(6) **(4+4+4+4 marks)** Find $f'(x)$ if: (Do not simplify your answers)

(a) $f(x) = \tan(x \cos x) + e^{10} + \log_4(3x^2 + 5x)$

(b) $f(x) = \left(\frac{4x^2 - 2x}{x^5 + 2x^3 + 1} \right)^{10}$

(c) $f(x) = \sqrt{1 - 25x^2} \arcsin(5x)$

(d) $f(x) = (\cos(2x))^{\sin x}$

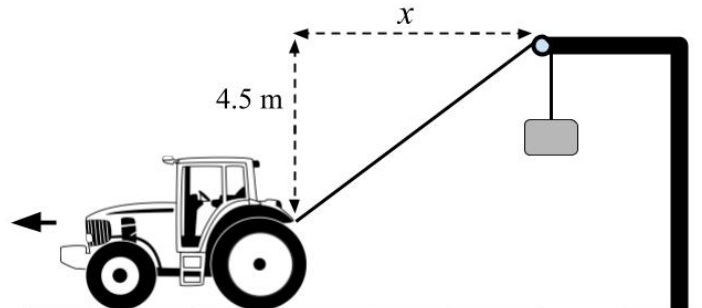
(7) **(4 marks)** If $f(x) = xe^x$ then find $f^{(100)}(x)$.

(8) **(5 marks)** If $y^2 e^{x^2-16} - \frac{x}{y} = 2$ then write an equation of the tangent line to the graph of the curve at $P(4, 2)$

(9) **(1+1+1 marks)** Decide if the following statements are true or false. If the statement is true give a justification and if it is false give a counterexample.

- (a) If a function f is continuous then f is differentiable.
- (b) If f is discontinuous at $x = a$ then f has a vertical asymptote at $x = a$.
- (c) If f is differentiable on $[a, b]$ then f has an absolute maximum and absolute minimum on the interval $[a, b]$.

(10) **(5 marks)** A farmer's tractor, travelling at 3 m/s , pulls a rope of fixed length attached to a bale of hay through a pulley. Using the dimensions indicated in the picture below how fast is the bale rising when $x = 5 \text{ m}$?



(11) (4+4 marks) Evaluate, using L'Hôpital's Rule, the following limits:

(a) $\lim_{x \rightarrow 1} \left(\frac{1}{2e^{x-1} - 2} - \frac{1}{2x - 2} \right)$

(b) $\lim_{x \rightarrow \infty} \left(1 + \sin \left(\frac{2}{x} \right) \right)^{5x}$

(12) (12 marks) If $f(x) = \frac{6}{x} - \frac{6}{x^3}$ and if $f'(x) = \frac{18 - 6x^2}{x^4}$, $f''(x) = \frac{12x^2 - 72}{x^5}$ then:

(a) Find the x-intercepts and the y-intercept, if any.

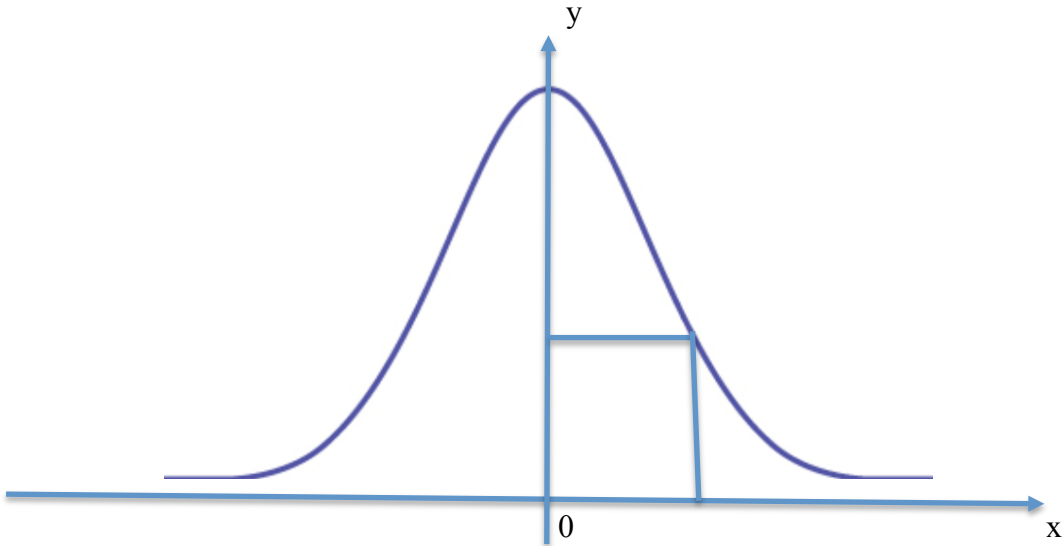
(b) Find all the horizontal and vertical asymptotes, if any. Justify your answers using limits.

(c) Find the intervals where the function is increasing and where it is decreasing. Find the points of local minimum and local maximum, if any.

(d) Find the intervals where the function is concave upward and concave downward. Find the inflection points, if any.

(e) Sketch the graph of the function and clearly label all the points on the graph.

(13) (5 marks) The rectangle shown in the figure has one side on the positive y-axis, one side on the positive x-axis and its upper right-hand vertex on the curve $y = e^{-x^2}$. What dimensions give the largest area and what is the area?



(14) (4+4 marks) Find:

(a) $\int \left(\frac{2}{3x^2} - x^5 + 5e^x + 4\sec^2 x \right) dx$

(b) $\int \frac{\sin(\ln x)}{3x} dx$

(15) (4 marks) Given the conditions $f''(x) = \frac{3x+1}{\sqrt{x}}$, $f(0) = 1$, $f(1) = 0$ find $f(x)$.

Answers:

1. (a) -1/16, (b) 0, (c) -1

2. $c=0$, limit=-1/3

3. $4x+1$

4. $x=1$, $x=-5$ non-removable discontinuities

5. $c=1/3$, $c=-1$

6. (a) $\sec^2(x \cos x) (\cos x - x \sin x) + \frac{1}{(3x^2 + 5x) \ln 4} (6x + 5)$

(b) $10 \left(\frac{4x^2 - 2x}{x^5 + 2x^3 + 1} \right)^9 \frac{(8x - 2)(x^5 + 2x^3 + 1) - (4x^2 - 2x)(5x^4 + 6x^2)}{(x^5 + 2x^3 + 1)^2}$

(c) $\frac{1}{2} (1 - 25x^2)^{-\frac{1}{2}} (-50x) \arcsin(5x) + \sqrt{1 - 25x^2} \frac{1}{\sqrt{1 - 25x^2}} (5)$

(d) $(\cos(2x))^{\sin x} \left(\cos x \ln(\cos(2x)) - \frac{2 \sin x \sin(2x)}{\cos(2x)} \right)$

7. $(100 + x)e^x$

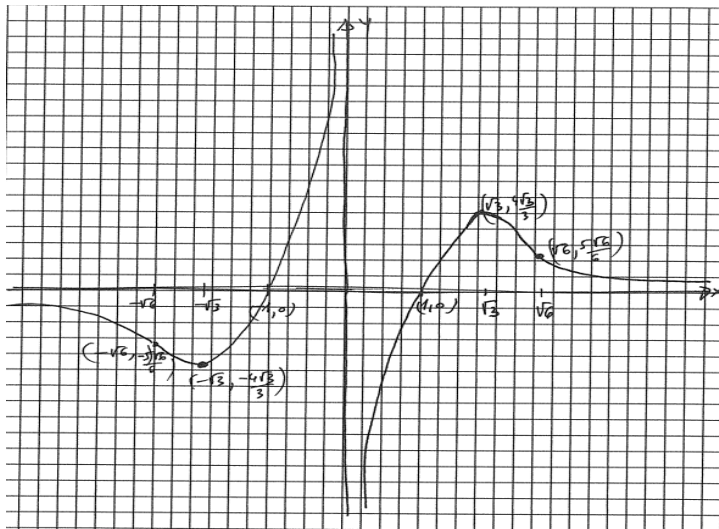
8. $y = -\frac{63}{10}x + \frac{136}{5}$

9. (a) False, (b) False, (c) True

10. 2.23m/s

11. (a) $-1/4$, (b) e^{10}

12.



13. $x = \frac{\sqrt{2}}{2}, y = e^{-\frac{1}{2}}, A = \frac{\sqrt{2}}{2} e^{-\frac{1}{2}}$

14. (a) $-\frac{2}{3x} - \frac{x^6}{6} + 5e^x + 4\tan x + C$, (b) $-\frac{\cos(\ln x)}{3} + C$

15. $\frac{4}{5}x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} - \frac{47}{15}x + 1$