Dawson College
Mathematics Department
FINAL EXAMINATION
Engineering Mathematics II
201-942-DW section 00001

December 13th, 2019

Instructor: Oxana Cerba

Time: 9:30-12:30

Instructions:

- Print your name and student ID number in the space provided on the Cover Sheet.
- All questions are to be answered directly on the examination paper in the space provided. Show your complete work and give explanations.
- ONLY SHARP EL-531X, XG or XT are permitted.

This examination consists of 14 **questions. Please ensure that you have a complete examination.**

This examination must be returned intact.

Question 1. [**9 marks**] Find the derivative of each function below. DO NOT SIMPLIFY

(a) $y = (7x^3 - \sqrt{x})^{14}$

(b) $y = \frac{x+2}{\sqrt{x^2-2x}}$

(c) $y = (x^2 - 1)^5(x - 3)^4$

Question 2. [**4 marks**] The angular displacement of a rotating body is given by $\theta = 18.5t^2 + 12.8t + 14.8\ rad$. Find (a) the angular velocity and (b) the angular acceleration at $t = 3.50s$.

Question 3. [**6 marks**] Given $\sqrt{xy} = 2x + y^2$. Using implicit differentiation find y'.

Question 4. [**6 marks**] Given $f(x) = \sqrt{4 + 3x}$.

(a) Using the definition of the derivative find $f'(x)$. (No points will be given for other method of solutions.)

(b) Find the equations of the tangent and normal lines at the point (4, 4).

Question 5. [**6 marks**] A point moves along the curve $x^2 - y^2 = 144$ with a horizontal velocity $v_x = 15.0\ cm/s$. Find the total velocity when the point is at (13.0, 5.0)

Question 6. [**8 marks**] A car leaves an intersection traveling west. Its position 4 sec later is 20ft from the intersection. At the same time another car leaves the same intersection heading north so that its position 4sec later is 28ft from the intersection. If the speed of the cars at that instant of time is 9 ft/sec and 11 ft/sec respectively, find the rate at which the distance between the two cars is changing.

Question 7. [**4+4 marks**] Given the function $f(x) = x^4 - 2x^2 + 4$

(a) Find max/min and intervals where the function is increasing /decreasing.

(b) Find inflection points and intervals of concavity.

Question 8. [**8 marks**] A manufacturer needs to make a cylindrical can that will hold 1.5 liters of liquid. Determine the dimensions of the can that will minimize the amount of material used in its construction. ($V_{cylinder} = \pi r^2 h, \quad A_{lateral} = 2\pi r h$)

Question 9. [**12 marks**]

(a) $\int x^7(1 + x^8)^{31} dx$

(b) $\int \frac{1+x^2}{\sqrt{x}} dx$

(c) $\int_{\sqrt{2}}^{3} x\sqrt{7+x^2}dx$

(d) $\int_{0}^{3} \frac{dx}{1+5x}$

Question 10. [3+4 marks]

(a) Find the area of the region bounded by the graph of $y = 2 - x^2$ and $y = x^2$

(b) Find the coordinates $\bar{x}$ and $\bar{y}$ of the center of mass of a thin plate covering the region from part (a)

Question 11. [6 marks] A canister is dropped from a helicopter 500 m above the ground. Its parachute does not open, but the canister has been designed to withstand an impact velocity of 100 m/s Will it burst? (Take $g = -9.8m/s^2$)

Question 12. [6 marks] Find the solution of the differential equation that satisfies the given initial condition $y' = \frac{y^2}{x}$, $y(1) = \frac{1}{2}$

Question 13. [6 marks] Find the exact length of the curve $y = \frac{x^3}{3} + \frac{1}{4x}$ for $1 \leq x \leq 2$

Question 14. [8 marks] A conical tank with height 10 m and base radius 4 m is full of water. Find the work needed to pump the water to a height of 6 m above the top of the tank. (The density of the water is 1000 kg/m^3)

Answers

Question 1 (a) $y' = 14(7x^3 - \sqrt{x})^{13}(21x^2 - \frac{1}{2}x^{-\frac{1}{2}})$ (b) $y' = \frac{\sqrt{x^2-2x} - (x+2)(x^2-2x)^{-\frac{1}{2}}(x-1)}{x^2-2x}$

(c) $y' = 10x(x^2-1)^4(x-3)^4 + 4(x^2-1)^5(x-3)^3$

Question 2 $\omega = 142.3 rad/s \quad \alpha = 37 rad/s^2$

Question 3 $y' = \frac{4\sqrt{xy} - y}{x - 4y\sqrt{xy}}$

Question 4 $y' = \frac{3}{2\sqrt{4+3x}} \quad y_{tangent} = \frac{3}{8}x + \frac{5}{2} \quad y_{normal} = -\frac{8}{3}x + \frac{44}{3}$

Question 5 $v = \frac{v_x}{cos\theta} = 41.67 cm/s$

Question 6 Using $x^2 + y^2 = z^2$ we have $\frac{dz}{dt} = 14.18 ft/s$

Question 7

(a) $f'(x) = 4x(x^2-1)$, f(x) is increasing on $[1,0] \cup [1, \infty[$ and decreasing on $]-\infty, -1] \cup [0,1]$ Points (-1,3) and (1,3) are minima, (0,4) is the maximum.

(b) $f''(x) = 4(3x^2-1)$, f(x) is concave up on $[-\infty, -\sqrt{\frac{1}{3}}] \cup [\sqrt{\frac{1}{3}}, \infty[$ and concave down on $[-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}]$. Inflection points at $x = -\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}$

Question 8 $r = 0.4887 dm \quad h = 2.077 dm$

Question 9

(a) $\frac{(1+x^8)^{32}}{256} + C$ (b) $2x^{1/2} + \frac{2}{5}x^{5/2} + C$

(c) $\frac{37}{3}$ (d) $\frac{\ln 16}{5}$

Question 10 $A = 8/3u^2 \quad \bar{x} = 0, \quad \bar{y} = 1$

Question 11 $v = 98.98 m/s$ It will not burst

Question 12 $-\frac{1}{y} = \ln|x| - 2$

Question 13 $\frac{59}{24}$

Question 14 $14 * 10^6 J$