Final Examination

Dawson College-Mathematics Department

201-912-DW: Applied Mathematics for Civil Technology – Fall 2019

December 20, 2019 9:30-12:30

Examiners: S. Soltuz, O. Veres

Name: _____

Student ID: _____

- Print your name and student ID number in the space provided above.
- All questions are to be answered directly on the examination paper in the space provided. If you need more space for your answer use the back of the page.
- Books, notes, cell phones, or any electronic devices are NOT permitted. Dictionaries are permitted. Only Sharp EL-531XG, EL-531X, or EL-531XT calculators are permitted.
- SHOW ALL YOUR WORK clearly and justify all your answers.
- Verify that your final examination copy has a total of 17 problems and 16 pages including this cover page and the **last page with formulas.**
- DO NOT TEAR ANY PAGE OFF. You have to submit your examination booklet intact.

1. [5 marks] Given two points $P(-6\,,9)$ and $Q(a^2-5a\,,\ a^2)$
   a. Find the **slope** of the line passing through these points. **Simplify your answer**.

Ans: $m=\frac{a+3}{a-2},\ \ a\neq 2$

   b. Find $\boldsymbol{a}$, if the slope of the line that is **perpendicular** to the line through the points $\boldsymbol{P}$ and $\boldsymbol{Q}$ is $\frac{4}{5}$

Ans: $a=-\frac{2}{9}$

2. [5 marks] Given the quadratic function $f(x)=2x^2-4x+1$.
   a. Find the $x-$ and $y-$ intercepts of the graph.

Ans: $\left(\frac{2\pm\sqrt{2}}{2}\ ,0\right)$; $(0,1)$

b. Find the coordinates of the vertex of the graph.

Ans: (1,-1)

c. Convert the given general form of the function into the standard form.

Ans: $y = 2(x-1)^2 - 1$

d. Find the domain and range of the function. Determine whether the function yields a relative and absolute maximum or minimum and find it.

Ans:R, $[-1, \infty)$, $f(1) = -1$ is minimum

e. Sketch the graph of the function.

3. [5 marks] An earthquake with an **intensity** of **I** has a Richter Scale **magnitude** of

$$\boldsymbol{M = log\left(\frac{I}{I_0}\right)}$$

where $\boldsymbol{I_0}$ is the **measure of a zero- level** earthquake.
Find the intensity of the 1999 Taiwan earthquake, which measured **7.6** on the Richter scale.
(Hint: find I in terms of I_0)

Ans:$\mathrm{I} = \mathrm{I}_0 \cdot 10^{7.6}$

4. [5 marks] Solve for t

$$3^{1-4t} = 5^{t+3}$$

Ans: $t = \frac{\ln 3 - 3\ln 5}{4\ln 3 - \ln 5}$

5. [5marks] Find the **domain** and the **inverse** of $\boldsymbol{f(x) = 3\log_5(x-2)+1}$

Ans: $(2,\infty)$, $f^{-1}(x) = 5^{\frac{x-1}{3}} + 2$

6. [5 marks] Given the formula for stress in a rectangular beam, solve for $\boldsymbol{d}$:

$$c = \frac{bd}{\sqrt{b^2+d^2}}$$

Ans: $d = \pm\sqrt{\frac{b^2c^2}{b^2-c^2}}$

7. [6marks] Solve the system

$$\begin{cases} 9x + 3y + z = -4 \\ x + y + z = -2 \\ 4x + 2y + z = -2 \end{cases}$$

Ans: (-1,3,-4)

8. [5+5 marks] Given $f(x) = \frac{2+x}{1-x}$ and $g(x) = \frac{1}{x-5}$

a. Find $(\boldsymbol{fog})(\boldsymbol{x})$, **simplify** your answer completely, then find the **domain** of it.

Ans: $\frac{2x-9}{x-6}, x \neq 5,6$

b. Find $\boldsymbol{f^{-1}(x)}$

Ans: $\frac{x-2}{x+1}$

9. [5 marks] Solve the equation for x: $x - \sqrt{x + 15} = 5$

Ans: x=10

10. [3+2 marks]
    a. **Simplify** the numerator, then **rationalize** the denominator and **simplify** your answer.

$$\frac{\sqrt{50} - \sqrt{18} - \sqrt{2} + \sqrt{20} - \sqrt{5}}{\sqrt{5} - \sqrt{2}}$$

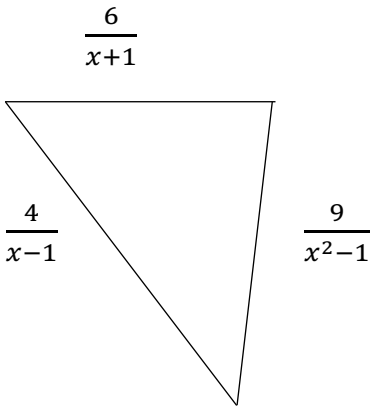
Ans: $\frac{7+2\sqrt{10}}{3}$

    b. Simplify the exponential expression and write your answer with **positive exponents**

$$(-8\,xy)^{5/3} y^{1/3} x^{-2/3}$$

Ans: $-32xy^2$

11. [6 mark] Find x such that the perimeter of the triangle is 9 cm. Note that the figure is not proportional.



Ans: x=2

12. [5 +4 marks]

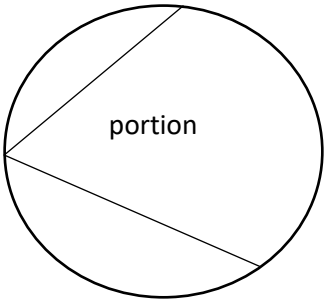
a. Show that

$$\frac{1-\cos^4\theta}{1-\cos^2\theta} = 2-\sin^2\theta$$

b. Find the **exact** value of $\cos\frac{2\pi}{3} - \tan\frac{\pi}{3} + \csc\frac{\pi}{2} + \sin\frac{4\pi}{3}$

Ans: $\frac{1-3\sqrt{3}}{2}$

13. [7 marks] Find the area of the portion if the cords are 4 cm and 5cm, respectively and the radius of the circle is 3 cm.



Ans: $A = 21.26\ cm^2$

14. [5 marks] In an early spAring the temperature T was measured in a small city during 5 working days with the results in the table below, showing the highest temperature on each day.
    a. Using the **formulas given on the last page**, find the **least squares regression line** for T as a function of the day t, starting with Monday, the 1st day. Show all your work.

$t(day)$	1	2	3	4	5
T(°C)	3	2	0	2	4

Ans: T=1/5 t+ 8/5

    b. What is the highest temperature that one can predict for Sunday, the 7th day?

Ans: $T = 3$°C

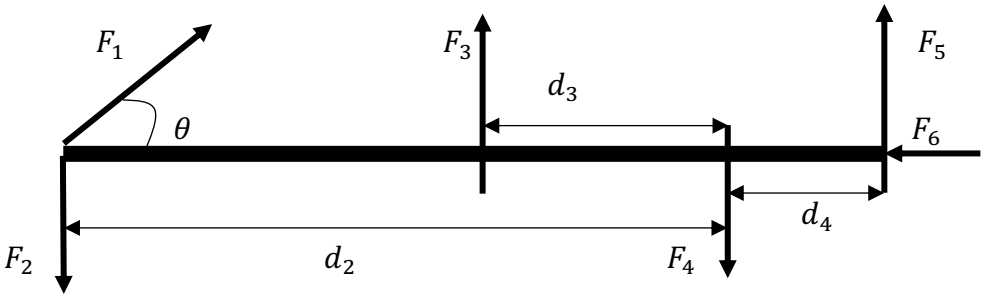
15. [5 marks] Solve the trigonometric equation

$2\sin^2 x - \sin x = 1 \qquad 0 \leq x \leq 360°$

Ans: $90°, 210°, 330$

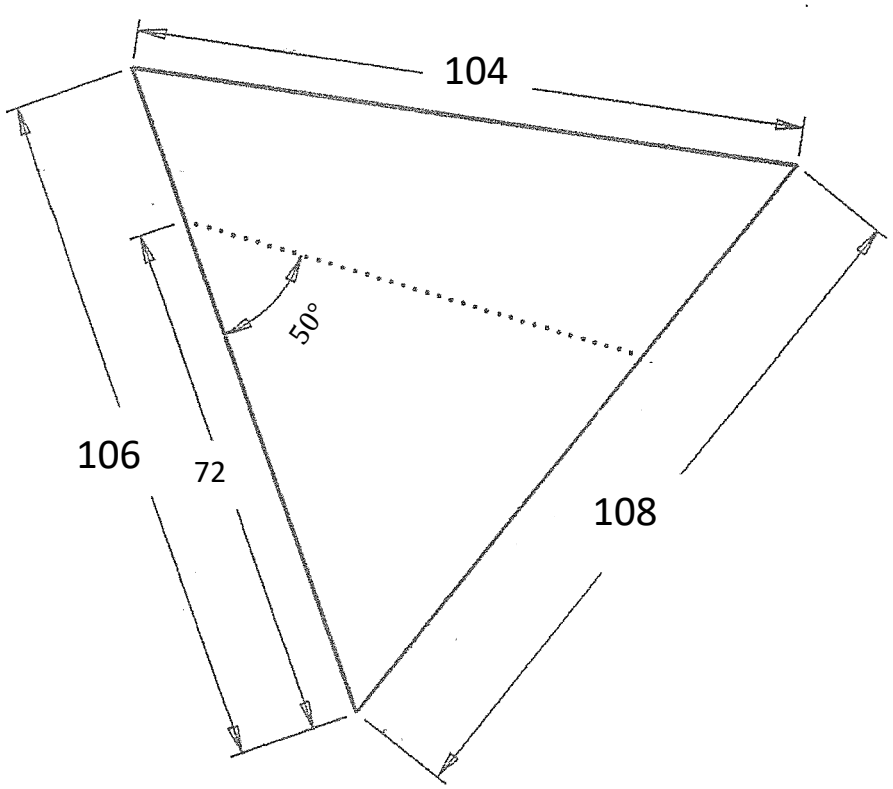
16. [5 marks] The following are in mechanical equilibrium. Find the missing forces and distances. In the diagram the distances are not proportional.

$F_4 = 6N\ , F_5 = 4N, F_6 = 2N, \theta = 45°, d_2 = 5m, d_3 = 2m, d_4 = 1m$



Ans: $F_1 = 2\sqrt{2}\ N, F_2 = F_3 = 2N$

17. [7 marks] Given the figure below, solve the small triangle.



Ans:$A = 58.14°,\ C = 71{,}86°,\ AC = 58{,}04,\ BC = 64{,}35$

Equation of the least-squares regression line: $y = ax + b$ where

$$a = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2} \qquad b = \frac{(\sum x^2)(\sum y) - (\sum xy)(\sum x)}{n\sum x^2 - (\sum x)^2}$$

OR

$$a = \frac{\frac{\sum xy}{n} - \overline{x} \cdot \overline{y}}{\frac{\sum x^2}{n} - (\overline{x})^2} \qquad b = \overline{y} - a\overline{x} \qquad \overline{x} = \frac{\sum x}{n} \qquad \overline{y} = \frac{\sum y}{n}$$

$$\sin(a + b) = \sin \text{a} \cos b + \cos a \sin b$$

$$\sin(a - b) = \sin \text{a} \cos b - \cos a \sin b$$

$$\cos(a + b) = \cos \text{a} \cos b - \sin a \sin b$$

$$\cos(a - b) = \cos \text{a} \cos b + \sin a \sin b$$

$$\cos(2a) = \cos^2 a - \sin^2 a$$

$$\sin(2a) = 2 \sin a \cos a$$